

CHAPTER 15 LECTURE – RISK AND INFORMATION

Probability and Expected Value:

Expected value: the sum of all possible outcomes, weighted by its respective probability of occurrence.

In addition to the expected value of a gamble, most people also consider how they feel about each of its possible outcomes.

People choose the alternative that has the highest **expected utility**.

Expected utility: the expected utility of a gamble is the expected value of utility over all possible outcomes.

The expected values of the outcomes of a set of alternatives need not have the same ranking as the expected utilities of the alternatives.

We need to extend the model of decision making by individuals and firms to include *uncertainty*. When uncertainty can be quantified, it is sometimes called *risk*.

Definition: A *lottery* is any event with an uncertain outcome.

It describes a situation where the outcome (state of nature) is not sure but the likelihood of the event can be quantified.

Definition: A *probability* of an outcome (of a lottery) is the likelihood that this outcome occurs.

- The probability of any particular outcome is between 0 and 1
- The sum of the probabilities of all possible outcomes equals 1 (outcomes are exhaustive)

(a) *Frequency:*

- the historical frequency of the outcome can be an estimate of the probability with which it will occur $P = n/N$

(b) *Subjective Probability:*

- when we do not have a history that allows us to calculate the frequency, we just use whatever information we have to form subjective estimates of the likelihood.

(c) *Probability Distribution* relates the probability of occurrence to each possible outcome.

Definition: The *expected value* of a lottery is a measure of the average payoff that the lottery will generate.

$$EV = \Pr(A)A + \Pr(B)B$$

Where: $\Pr(\cdot)$ is the probability of (\cdot) A, and B are the payoffs if outcome A or B occurs.

Example:

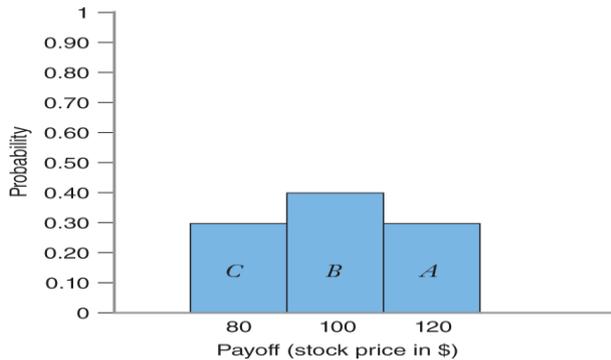
Let $\Pr(A) = 0.5$ and $\Pr(B) = 0.5$ Outcome of A is \$10 and Outcome of B is \$0

$$EV = 0.5 (\$10) + 0.5 (\$0) = \$5.00$$

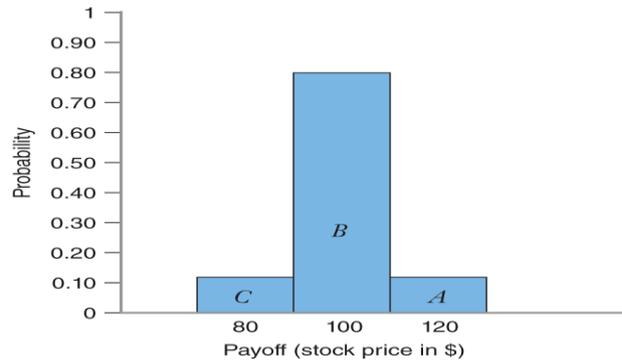
A fair gamble would imply you would be willing to pay \$5.00 for a ticket.

The Concept of Variance

Refer to [Random Variables: Mean, Variance and Standard Deviation](#)



(a) Internet company



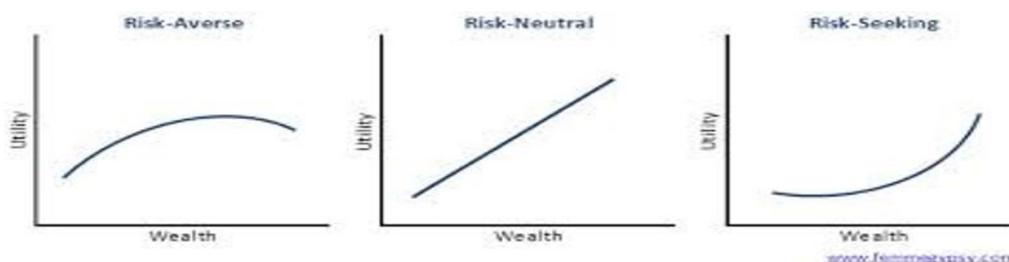
(b) Public utility company

Probability Distributions, Riskiness, and Variances

The riskiness of investing in the Internet company is much greater than the riskiness of investing in the public utility company. The probability that the actual outcome will differ from the expected outcome (outcome B in both cases) is 6 in 10 for the Internet investment but only 2 in 10 for the public utility investment. This is reflected in the difference in the variances (\$240 for the Internet investment and \$80 for the public utility investment).

Risk Preferences

- The risk preferences of individuals can be classified as follows:
 - An individual who prefers a sure thing to a lottery with the same expected value is **risk averse**.
 - An individual who is indifferent about a sure thing or a lottery with the same expected value is risk neutral.
 - An individual who prefers a lottery to a sure thing that equals the expected value of the lottery is **risk loving (or risk preferring)**



Risk Averse Person

Flip a coin. Win \$30 if heads and lose \$30 if tails.

$EV = 0.5(-\$30) + 0.5(\$30) = 0$. If it costs nothing to play this is a fair gamble.

Now a person has initial wealth of \$40.

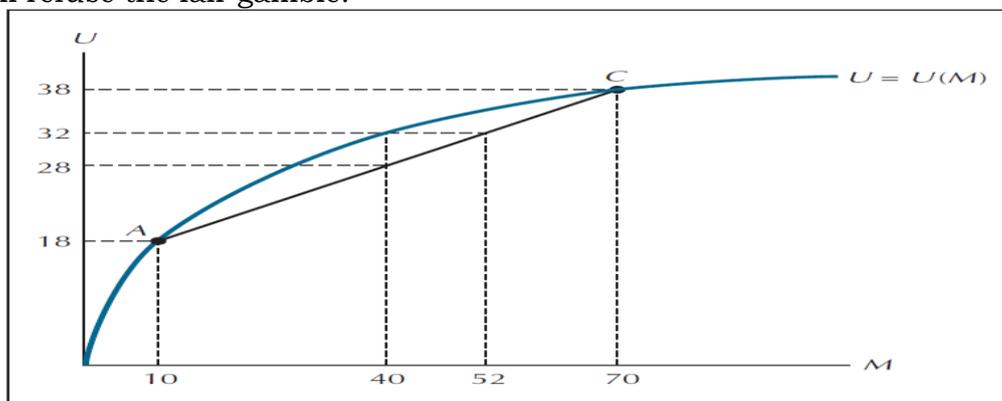
$EV \text{ of wealth is } 0.5(\$40 - 30) + 0.5(\$40 + \$30) = \$40$ Again, we note this is a fair gamble.

Look at the diagram below which shows utility for various levels of wealth.

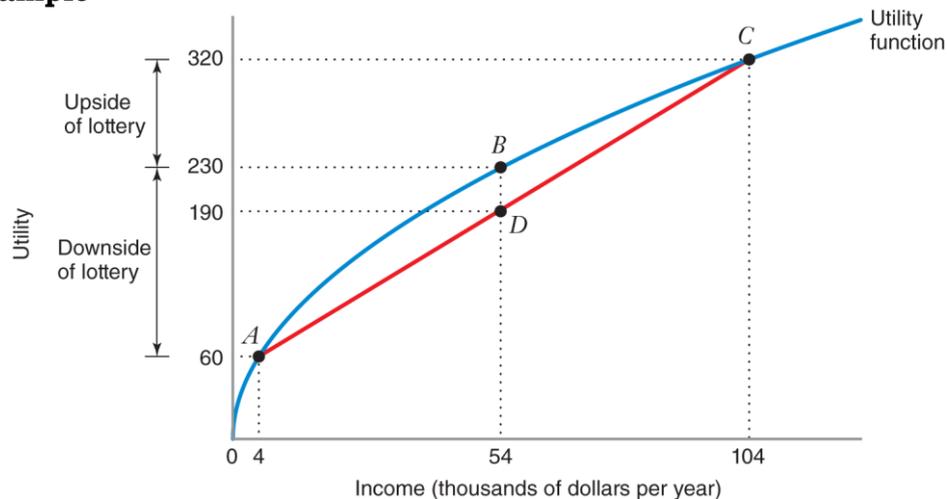
What is Expected Value of Utility?

$EU = 0.5U(\$40 - \$30) + 0.5U(\$40 + \$30) = 0.5U(\$10) + 0.5U(\$70) = 0.5U(18) + 0.5U(38) = 28$

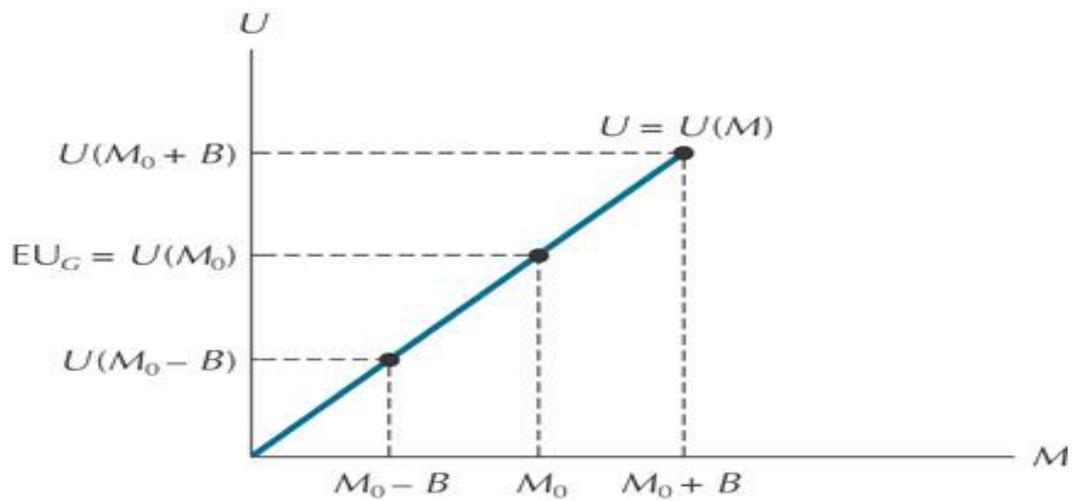
If you don't gamble your income is \$40 which yields utility of 32. Thus, a risk averse person will refuse the fair gamble.



Another Example

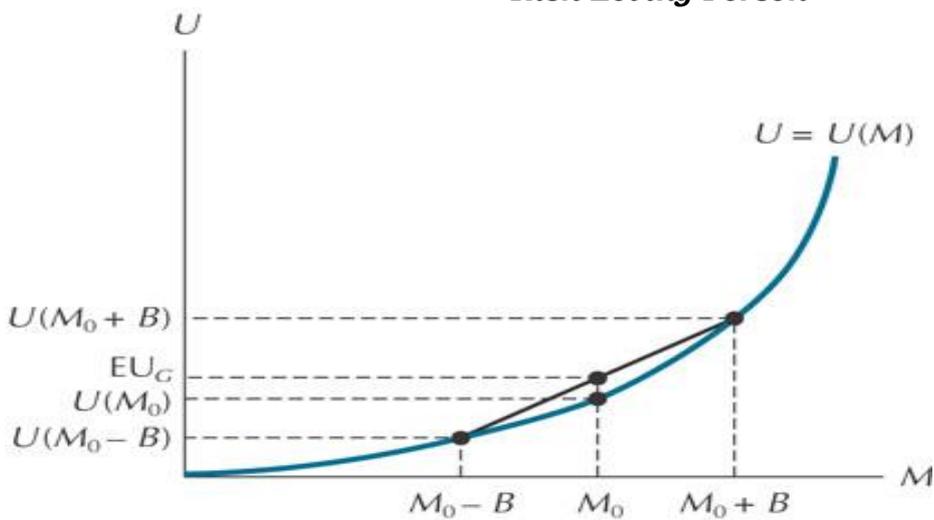


Utility Function and Expected Utility for Risk Averse Person: Your utility if you take the job with the established company will be 230 (point B). If you take the job with the start-up, there is a 0.50 probability that your utility will be 320 (point C, if you earn \$104,000) and a 0.50 probability that your utility will be 60 (point A, if you earn \$4,000), yielding an expected utility of 190 (point D). Because your utility with the established company is greater than your expected utility with the start-up company, you will prefer the offer from the established company.

Risk Neutral Person

$$EU = (1/2)U(M_0 - B) + (1/2)U(M_0 + B) = U(M_0)$$

$$EV = (1/2)(M_0 - B) + (1/2)(M_0 + B) = (M_0) \quad \text{Consumer is indifferent.}$$

Risk Loving Person

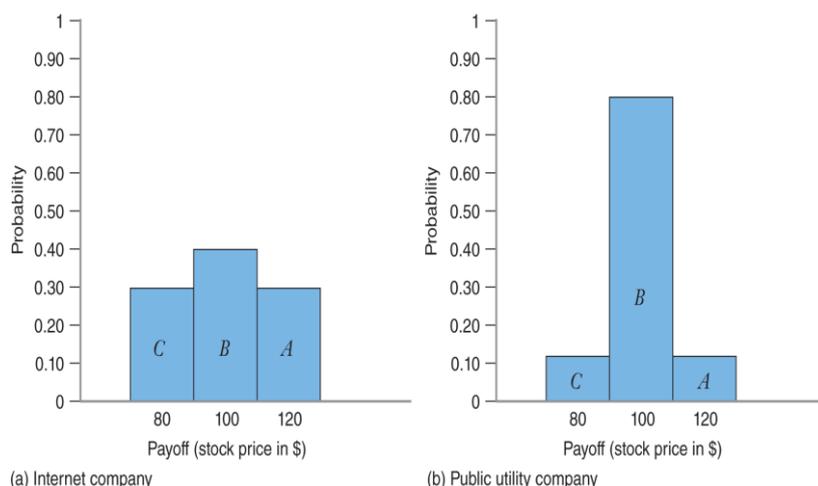
$$EU_{\text{Gamble}} = (1/2)U(M_0 - B) + (1/2)U(M_0 + B) = U(M_0)$$

$$EV = (1/2)(M_0 - B) + (1/2)(M_0 + B) = (M_0)$$

In this case $EU_{\text{Gamble}} > U(M_0)$ Choose to gamble.

Sample Problem: Computing the Expected Utility for Two Lotteries: Risk-Neutral and Risk-Loving Decision Makers

Suppose two decision makers are each considering the investments in the two lotteries. One decision maker is risk neutral, with the utility function $U(I) = (100)I$, while the other is risk loving, with the utility function $U(I) = (100)I^2$, where I denotes the payoff of the lottery.



a) Which lottery does the risk-neutral decision maker prefer?

Solution:

Remember $U(I) = (100)I$

Expected utility of investing in Internet stock = $0.30(8,000) + 0.40(10,000) + 0.30(12,000) = 10,000$

Expected utility of investing in public utility stock = $0.10(8,000) + 0.80(10,000) + 0.10(12,000) = 10,000$

Since the two investments have the same expected utility, the risk-neutral decision maker is indifferent between them.

b) Which lottery does the risk-loving decision maker prefer?

Remember $U(I) = 100I^2$

Expected utility of investing in Internet stock

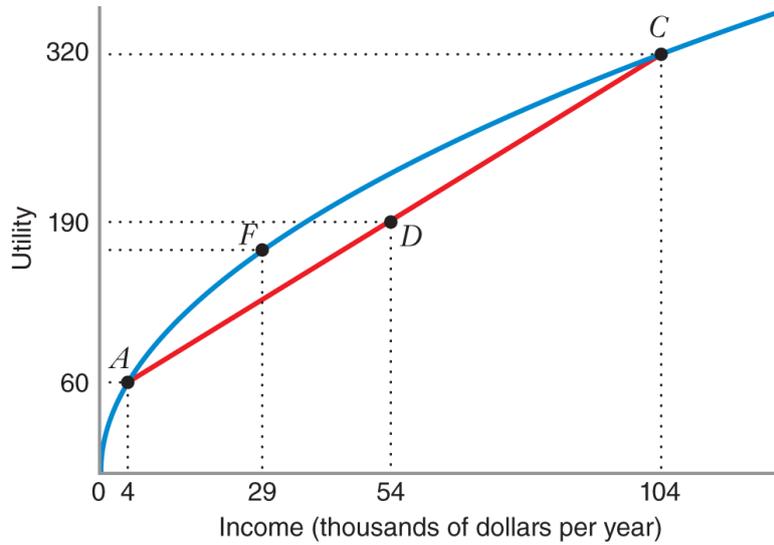
$$= 0.30(100)(80^2) + 0.40(100)(100^2) + 0.30(100)(120^2) = 1,024,000$$

Expected utility of investing in public utility stock

$$= 0.10(100)(80^2) + 0.80(100)(100^2) + 0.10(100)(120^2) = 1,008,000$$

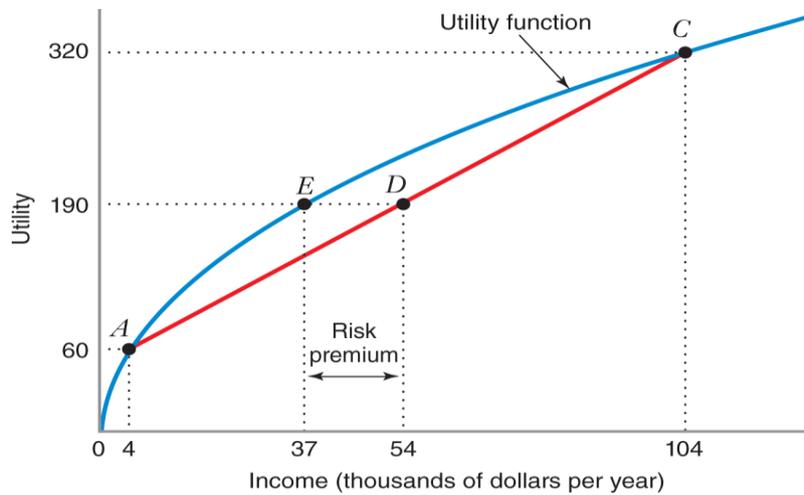
The risk-loving decision maker will prefer investing in the Internet stock, since the expected utility is higher than it is for investing in the public utility stock. This illustrates a general point. If lotteries L and M have the same expected value, but lottery L has a higher variance than lottery M, a risk-loving decision maker will prefer L to M.

Risk Premium: The amount of money that a risk-averse person would pay to avoid taking a risk.



A Risk-Averse Decision Maker Might Prefer a Lottery to a Sure Thing

If the salary offer from the established company were only \$29,000 per year, your expected utility from the start-up company's offer (point D) would exceed the utility from the established company's offer (point F). In this case, you would prefer the lottery to the sure thing.



The Risk Premium for a Risk-Averse Decision Maker: If the salary offer from the established company were \$37,000 per year, you would be indifferent between the start-up company's offer and the established company's offer because the two offers would have the same utility (190). The risk premium is given by the length of line segment ED, which equals \$17,000.

Bearing and Avoiding Risk

1) "Just Say No"

The simplest way to avoid risk is to abstain from optional risky activities.

2) Diversify

An investor *diversifies* by undertaking many investments (rather than one). This is also called risk pooling.

If the probability of a second event occurring falls when a first event occurs, the events are *negatively correlated*.

Similarly, if the second event becomes more likely when the first event occurs, the events are *positively correlated*.

If the probability of the second event occurring does not change with the occurrence of the first event, the events are *independent*.

3) Decision Tree and the Value of Information

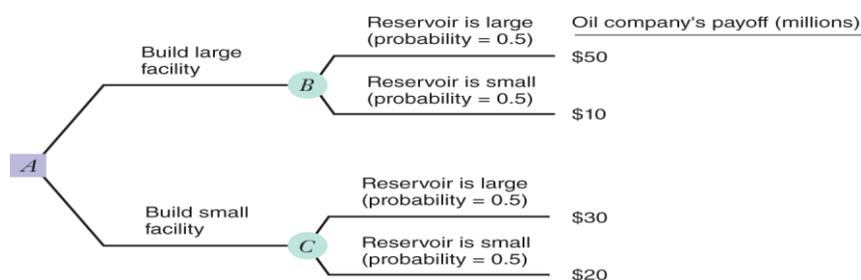
Question: How will a decision-maker choose a plan of action in the face of risk?

A *decision tree* is a diagram that describes the options available to a decision-maker, as well as the risky events that can occur at each point in time.

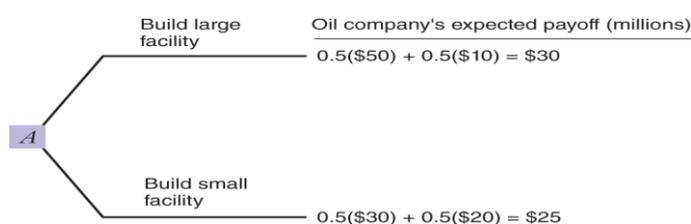
Example:

Decision Tree for Oil Company's Facility Size Decision

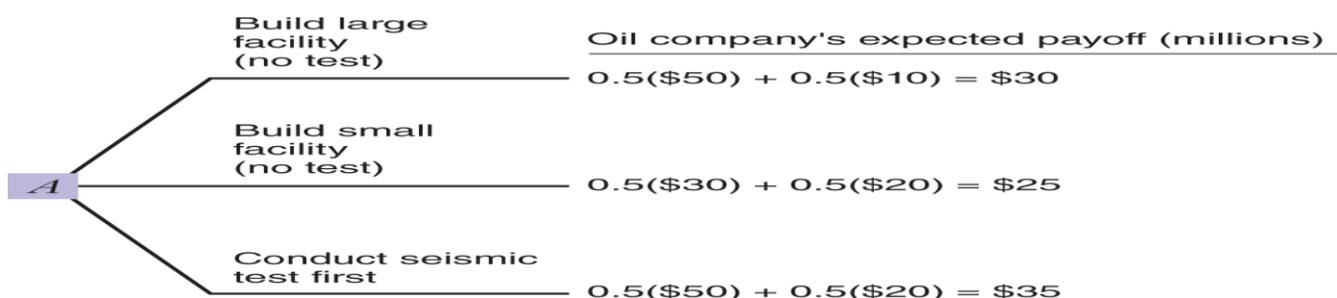
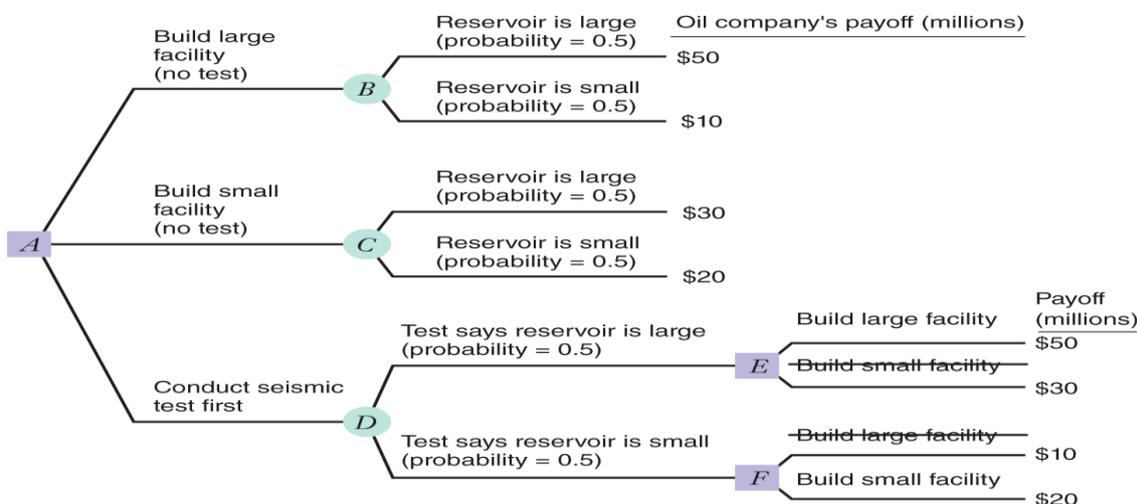
At decision node A, the company has two choices—build a large facility or build a small facility. At chance nodes B and C, the company faces lotteries with two possible outcomes (the reservoir is large or the reservoir is small, each with a probability of 0.50). The company's payoff (i.e., its profit) depends on its decision at node A and the actual outcome. A firm discovers an undersea oil field and decides whether to build a large capacity drill or a small capacity one. The oil reserves can be massive but they can also turn to be small...



Payoff: Build Large



DECISION TREES WITH A SEQUENCE OF DECISIONS (EXPLAIN OUTCOME)



Looking at Insurance

⇒ A risk-averse person gives money to the insurance company in the good state of nature, and the insurance company transfers money to the policyholder in the bad state of nature.

"Good" is defined also with respect to the "state" of nature in which it occurs (time, geography...).

If the state of nature is random, then insurance is a way of choosing the allocation consumption across the states!

Example: Fire insurance

Without insurance: \$50,000 if no loss or \$10,000 if loss

With insurance: \$50,000 - cost of policy (**no fire**) or

\$10,000 - cost of policy + value of Policy (\$40,000) (**fire**)

If Policy pays full compensation the compensation must be equal to \$40,000.

If the policy costs \$4000 will the person insure?

Let Utility = $U = I^{\frac{1}{2}}$ I = Income

You can show that this function exhibits diminishing marginal utility of income.

Let probability of fire be equal to 0.1

Expected income without insurance is $E(I) = 0.9(50,000) + 0.1(10,000) = 46,000$.

Certain Income with insurance is \$46,000 (policy costs \$4000)

In both cases income is the same. Will he insure?

$U(\text{Certain Income}) = U = I^{\frac{1}{2}} = U \sqrt{46,000} = 214.5$

Expected Utility = $0.9\sqrt{50,000} + 0.1\sqrt{10,000} = 211.3$

Person will insure since Utility is higher for certain income than expected utility of uncertain income.

If the policy is actuarially fair than cost = expected benefit payment or cost = $0.1(40,000) = 4000$

What would be the risk premium? Look at what would be the certain income required to give the individual the same utility as the expected utility.

We see Expected Utility = 211.3 or $I = 44,647.69$

We know $U = I^{1/2}$

Let RP = risk premium We solve $(50,000 - RP)^{1/2} = 211.3$

Solving for RP = $50,000 - RP = 44,647$ or $RP = 5,653$

Additional Material

Signaling: communication that conveys information.

Two properties of **signaling:** between potential adversaries:

1. Signals must be costly to fake- **Costly-to-Fake Principle**
2. If some individuals use signals that convey favorable information about themselves, others will be forced to reveal information even when it is considerably less favorable. **Full-disclosure principle**

Costly-to-Fake Principle

Signalling occurs when it is hard to distinguish "good" types from "bad" types, and there is a costly method available to signal you are good type such that the cost of signalling is too high for bad types that they won't do it.

Examples: A firm has an initial public offering of stock. If the owner retains a large proportion of stock, this signals he believes the firm will be very profitable.

Bad firms won't be able to mimic this signal because it is too costly to retain the junk stock.

Dupont is a huge chemical firm. In the 1980's they built a huge capacity of titanium oxide plants to show that their cost of producing was so low they could sell to the whole market and would do just fine in a price war, their competitors dropped out.

A restaurant might locate itself in an expensive area where rent is very hard signaling it will be very successful (else it wouldn't be able to cover its high rent).

Economic applications of Costly to Fake Principle:

- **Product Quality Assurance**
- **Choosing a Trustworthy Employee**
- **Choosing a Hard-Working, Smart Employee**

The Full-disclosure Principle

Full-disclosure principle: individuals must disclose even unfavorable qualities about themselves, lest their silence be taken to mean that they have something even worse to hide.

Applications of Full Disclosure Principle

- **Product Warranties** - Producers know much more than consumers about how good their products are.
- **Regulating the Employment Interviewer**
- **The Stigma of the Newcomer** – Movers
- **The Lemons Principle** - Cars offered for sale, taken as a group, are simply of lower average quality than cars not offered for sale.

Asymmetric Information - A situation in which one side of the market—either buyers or sellers—has better information than the other.

Market for Lemons Example

<http://www.youtube.com/watch?v=N78gTX7VOwM>

Nice simple mathematical example of how asymmetric information can force markets to unravel. Attributed to George Akerlof, Nobel Prize a few years ago.

Problem Setup

- Market for used cars
- Sellers know exact quality of the cars they sell
- Buyers can only identify the quality by purchasing the good

Buyer beware: cannot get your \$ back if you buy a bad car

- Two types of cars: high and low quality
- High quality cars are worth \$20,000, low are worth \$2,000

Suppose that people know that in the population of used cars that $\frac{1}{2}$ are high quality. This is a strong (unrealistic) assumption and one that is not likely satisfied. Remember, buyers do not know the quality of the product until they purchase

How much are they willing to pay?

Expected value = $(1/2)\$20,000 + (1/2)\$2,000 = \$11,000$

People are willing to pay \$11K for an automobile

- Would \$11,000 be the equilibrium price?
- Who is willing to sell an automobile at \$11,000?
- High quality owner has \$20,000 auto
- Low quality owner has \$2,000

Only low-quality owners enter the market

Now suppose you are a buyer, you pay \$11,000 for an auto and you get a lemon, what would you do?

You would sell it for on the market for \$11,000. Eventually what will happen?

Low quality cars will drive out high quality and equilibrium price will fall to \$2000 so only low-quality cars will be sold.

Some solutions?

- Dealers can offer money back guarantees. Does not solve the asymmetric info problem, but treats the downside risk of asymmetric information.
- Buyers can take to a garage for an inspection
- Carmax

Health Insurance: Adverse Selection vs. Moral Hazard

How important is health insurance to you? Perhaps the answer to this question depends on your overall health condition. However, at some point you have to consider whether you want to base the decision of purchasing a health insurance plan solely on the current status of your health. The reality of it is that health insurance is a significant factor that affects your life and yet not everyone tends to feel this way. This is where the adverse selection verses moral hazard of health insurance controversy comes into play.

Adverse Selection of Health Insurance Plans

You may be wondering what exactly adverse selection is all about. Well, in terms of health insurance, adverse selection explains the inclination for you to only purchase health insurance if you know you will ultimately benefit from it. The way adverse selection works is that if you're in poor health, you have more of a tendency to buy a health insurance plan because you're aware of all the medical bills you will have to handle. On the flip side of

adverse selection, you may feel you are in fairly good health and therefore don't find the purchase of a health insurance plan essential. It makes sense to you because visiting the doctor once a year and paying a one-time fee is much more cost effective than making monthly health insurance payments.

If you think there is a connection between adverse selection and why health insurance companies check out your medical history during the screening process, you're absolutely correct! Prior to purchasing a health insurance plan, you are generally asked to fill out a lengthy medical history form, in which you answer specific health-related questions such as:

- Do you smoke?
- How much do you weigh?
- What kind of diseases/health conditions run in your family? (High blood pressure, diabetes etc.)
- Have you ever been treated for a terminal illness?
-

By gathering this type of information, health insurance companies are able to determine whether or not you will be a large financial liability to them. Unfortunately, based on that, you are often either denied coverage or charged with higher premiums/rates in order to balance the amount the health insurance company will be responsible to cover. On a brighter note, there are actually some insurance companies out there that will reward you for not smoking and being in good health by offering special discounts on your health insurance policy.

The Moral Hazard of Health Insurance

The moral hazard, as it relates to health insurance is a certain way of thinking about your medical costs concerning a health-related emergency when you know you are insured. In other words if you have an adequate health insurance plan, you're more inclined not to stress over payment in the case of a medical emergency, as you know your out-of-pocket expenses will be alleviated by your health insurance company. When it comes to the moral hazard of health insurance, you may also be less likely to keep up with annual physicals and other forms of preventative health care.

In the end, you have to think about the adverse selection and moral hazard of health insurance carefully. Even though you may be in good health now, you never know what the future holds for you and while you should remain positive, you also need to be prepared for unforeseen medical situations. If you already have a health insurance plan, you shouldn't take it for granted. While it may be tempting to neglect your routine check-ups and chalk it all up to the fact that if something should happen, you're covered: it's not a good idea. Your first priority should be your well-being and as long as you remember that, you won't get caught up in the controversy of adverse selection verses moral hazard!



Comments Anyone?