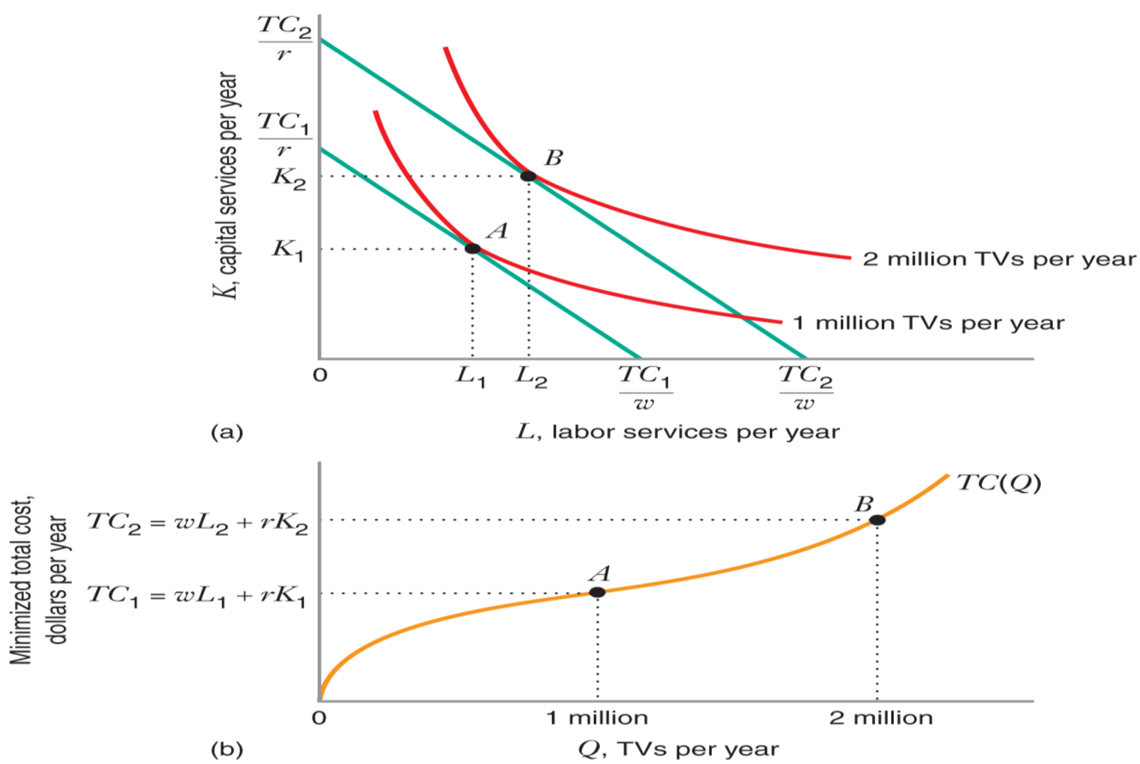
CHAPTER 8 LECTURE – COST CURVES

Long-Run Cost Curves

What we will now look at the long run -- this where a firm can adjust ALL inputs in an optimal manner. In consumer theory -- indifference curves -- the analogous problem is how you adjust in response to the prices of goods changing. If the prices of X and Y you adjust the combination of goods that you buy moving to another optimal bundle. Well, a firm does the same thing, it's buying labor and capital and if the prices of those things change, then a firm readjusts how many people it hires, how many computers to use, etc.

Let's take a firm that has the following production function: Q = f(K, L). And it can buy all the labor it wants at w and all the capital it wants at r.

Therefore, total costs are equal to: $TC = wL + rK$.



The long-run total cost curve shows how minimized total cost varies with output, holding input prices fixed and selecting inputs to minimize cost. Because the cost-minimizing input combination moves us to higher isocost lines, the long-run total cost curve must be increasing in Q. We also know that when Q = 0, long-run total cost is 0. This is because, in the long run, the firm is free to vary all its inputs, and if it produces a zero quantity, the cost-minimizing input combination is zero labor and zero capital. Thus, comparative statics analysis of the cost-minimization problem implies that the long-run total cost curve must be increasing in Q and must equal 0 when Q = 0.

Example:

A firm produces a product with labor and capital, and its production function is described by Q = LK. Suppose that the price of labor equals 2 and the price of capital equals 1. Derive the equations for the long-run total cost curve and the long-run average cost curve.

We see that the MP_L = K and the MP_K = L and based on our knowledge of production theory the firm will hire input such that. $\frac{MP_L}{MP_K} = \frac{w}{r} \; or \; \frac{K}{L} = \frac{2}{1} \; or \; K = 2L$.

TC = wL + rK or TC = 2L + 2L or TC = 4L Substituting K=2L into the production function yields Q = L2L = $2L^2$. We can then solve for L in terms of Q. $L = (0.5Q)^{\frac{1}{2}}$

And we can see $K = 2(0.5Q)^{\frac{1}{2}}$

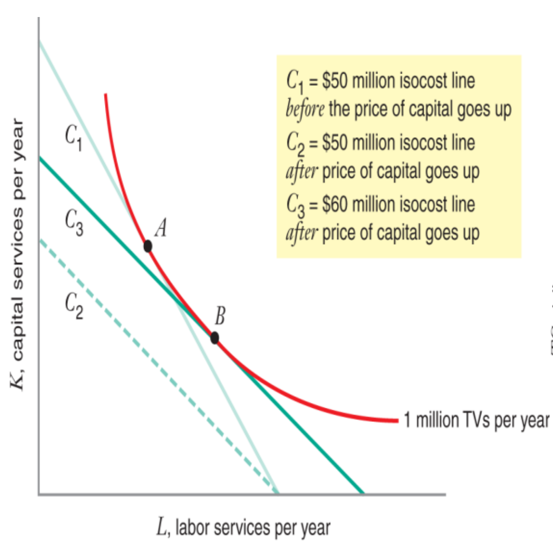
Plugging back into the cost function yields $TC = (2)(0.5Q)^{\frac{1}{2}} + 2(0.5Q)^{\frac{1}{2}} = 4(0.5Q)^{\frac{1}{2}}$

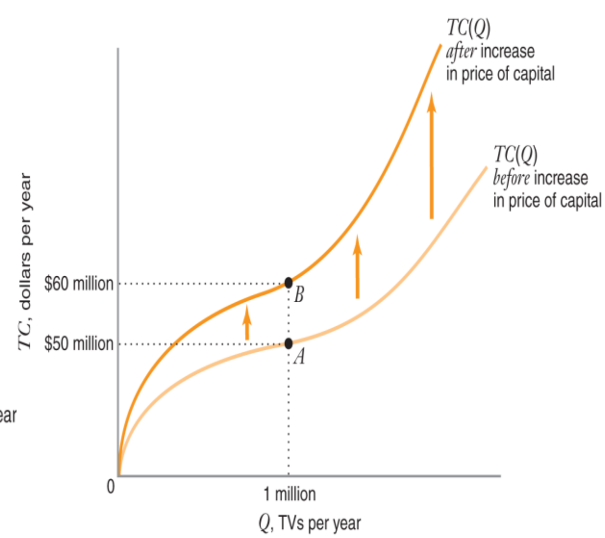
A little more math and we get $TC = 16^{\frac{1}{2}}(0.5)^{\frac{1}{2}}(Q)^{\frac{1}{2}} = (8Q)^{\frac{1}{2}}$

$$AC = \frac{(8Q)^{\frac{1}{2}}}{Q} = \left(\frac{8}{Q}\right)^{\frac{1}{2}}$$

LONG-RUN TOTAL COST CURVE SHIFT WHEN INPUT PRICES CHANGE?

Increase in the price of Capital





Short Run Cost Curves

Let's first review the important cost concepts:

Total Cost = TC

Total Fixed Cost = TFC

Total Variable Cost = TVC

Average Total Cost = ATC = TC/Q

Average Variable Cost = AVC = TVC/Q

Average Fixed Cost = TFC/Q

Marginal Cost = $\Delta TC/\Delta Q = \Delta TVC/\Delta Q = \frac{dTC}{dQ} = \frac{dTVC}{dQ}$

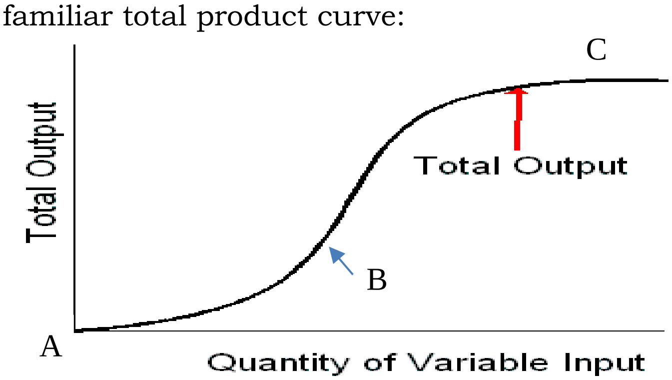
Total Cost is made up of two components in the short run: Total Fixed Cost and Total Variable Cost: TFC + TVC = TC.

TFC does not vary with the output rate. TFC is the cost of the fixed inputs. In the simple case that we examined where capital (K) was the input fixed in the short run.

TVC does vary with output rate. TVC is the cost of the variable inputs. In the simple case that we examined where labor (L) was the input that varied in the short run.

TVC = ωL TC = TFC + TVC

In the short run, then, we have some fixed inputs and some variable inputs that yield the familiar total product curve:



From AB let production function be $Q=L^2$

$dQ/dL=MP_L = 2L$.
$d^2Q/dL^2= 2 > 0$
Increasing marginal returns
$TVC = wQ^{1/2} = Q^{1/2}$ Let w = 1

$dTVC/dQ=\frac{1}{2}Q^{-1/2}$
$d^2TVC/dQ^2= - 1/4Q^{-3/2}<0$

Marginal Cost is downward sloping.

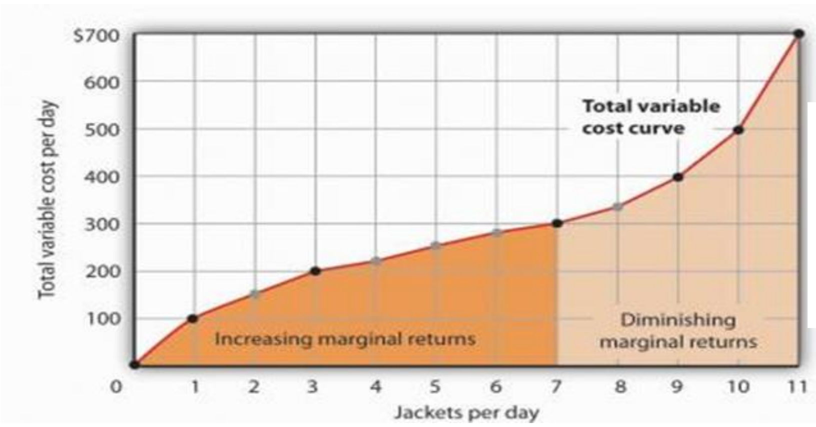
From BC let production function be $Q=L^{1/2}$

$dQ/dL= MP_L = \frac{1}{2}L^{-1/2}$
$d^2Q/dL^2= -1/4 L^{-3/2} < 0$
Decreasing marginal returns

$TVC = wQ^2 = Q^2$ Let w = 1
$dTVQ/dQ=2Q$

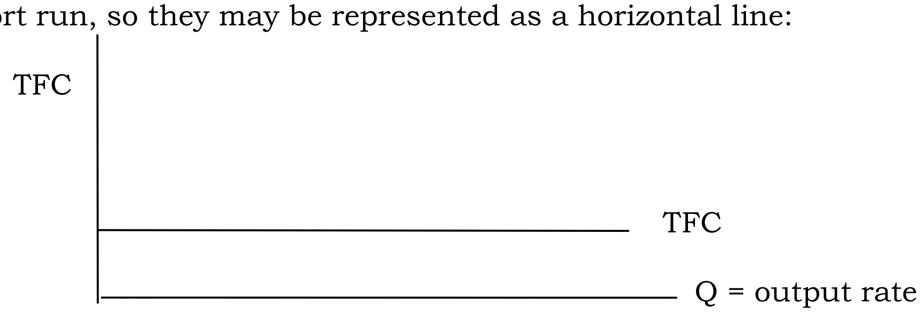
$d^2TVQ/dQ^2= 2 > 0$
Marginal Cost is upward sloping. TVC

It is easy to translate the quantity of the variable input into total variable cost (TVC): just multiply by the price of the input - the shape doesn't change. However, now we would like to express TVC as a function of the output of the product. This is easy: just flip the axes and we have the TVC curve:

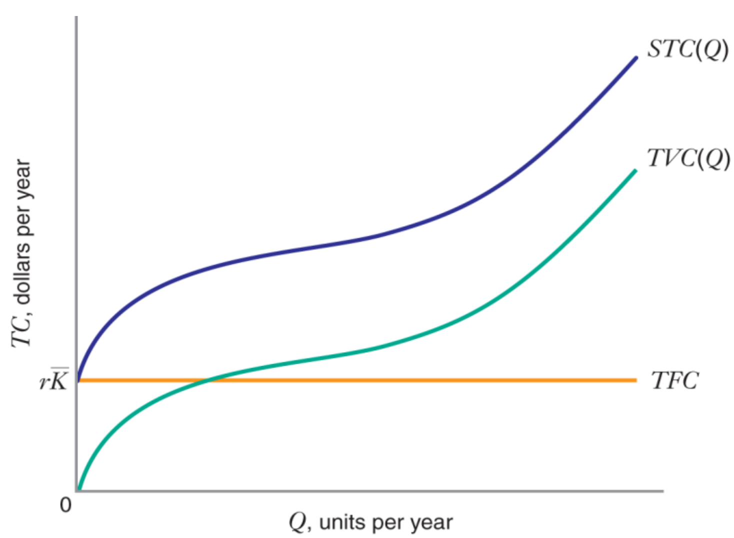


Total Variable Costs are the costs associated with hiring various levels of variable inputs in order to vary the rate of output in the short run."

Now we need to talk about Total Fixed Costs (TFC). The Fixed Costs do not change as output varies in the short run, so they may be represented as a horizontal line:



Finally, recall that the firm's Total Cost of producing output (TC) is the sum of its Total fixed cost (TFC) and it's Total Variable Cost (TVC):



But what the firm really needs to know is how the costs are distributed across the individual units of outputs and how they change when the level of output is increased or decreased. Thus, we want to look at the shapes of Average Total Cost (ATC), Average Variable Cost (AVC), Average Fixed Cost (AFC), and Marginal Cost (MC).

Average Fixed Cost (AFC) AFC = TFC / Q

Average Variable Cost (AVC)

AVC is the slope of the line from the origin to the point on the TVC function. This slope is a direct result of the law of diminishing marginal returns.

$AVC = TVC / Q = wL / Q$ For simplicity assume $w = 1$

But $AP_L = Q / L$ So $AVC = 1 / AP_L$.

As the AP_L fall, AVC rises and as AP_L rises, AVC falls. If AP_L is constant, AVC is constant.

Average Total Cost (ATC)

$ATC = TC / Q = (TFC + TVC) / Q = AFC + AVC$

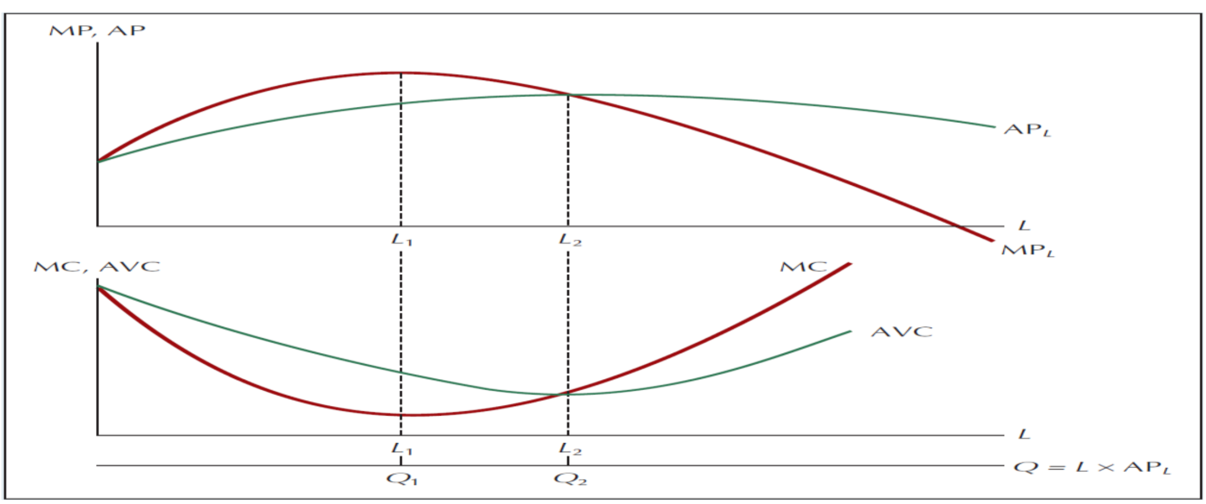
Marginal Cost (MC)

MC is the slope of TC. The shape is a direct result of the law of diminishing marginal returns.

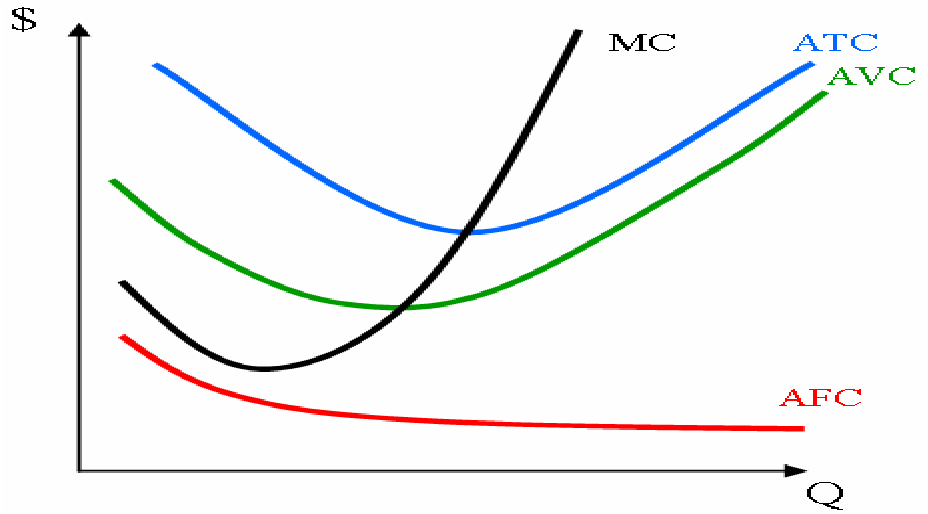
$MC = \Delta TVC / \Delta Q = w \Delta L / \Delta Q$ For simplicity assume $w = 1$

But $MP_L = \Delta Q / \Delta L$ So $MC = 1 / MP_L$.

The Relationship Between MP, AP, MC, and AVC



Consider the geometry of Average and Marginal Cost functions: Given the total cost function, we frequently want to derive the average and marginal cost functions. This can be done graphically in much the same way that we derived the **average** and **marginal** product curves. You can do this as an exercise. Your result should look like that below.



The Relationships Among Short-Run Cost Curves

1. ***AFC*** continuously declines and approaches both axes asymptotically.
2. ***AVC*** initially declines reaches a minimum and then increases.
3. When ***AVC*** is at a minimum it is equal to ***MC***.
4. ***ATC*** initially declines reaches a minimum and then increases.
5. When ***ATC*** is at a minimum it is equal to ***MC***.
6. ***MC*** is less than ***AVC*** and ***ATC*** when both curves are declining.
7. ***MC*** is greater than ***AVC*** and ***ATC*** when these curves are increasing.
8. ***MC*** equals ***AVC*** and ***ATC*** when both curves reach their minimum values.

MC passes through minimum of AC.

Proof:

$$\frac{dAC}{dQ} = \frac{d\frac{TC}{Q}}{dQ} = \frac{(Q\frac{dTC}{dQ} - TC\frac{dQ}{dQ})}{Q^2} = \frac{Q(MC) - TC}{Q^2} = \frac{MC - AC}{Q} = 0$$

When AC is minimized MC = AC

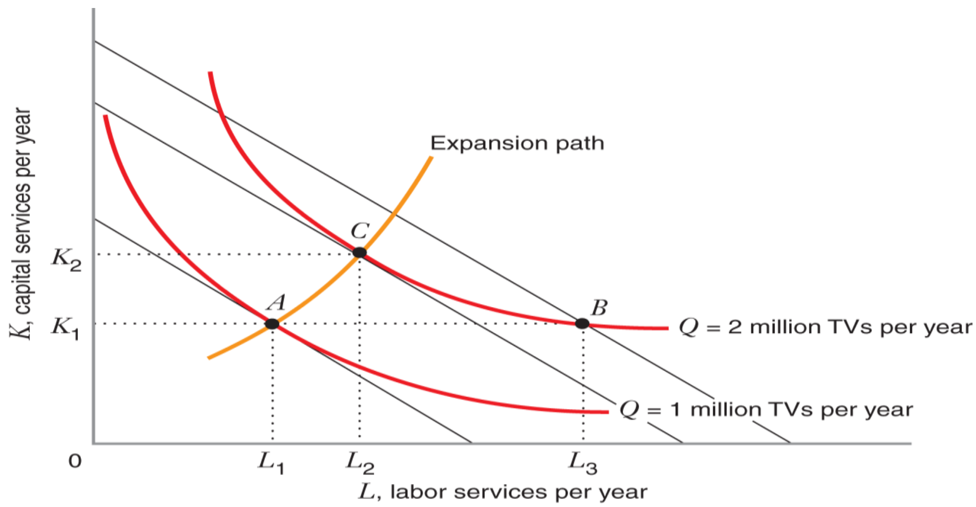
Intuitive argument:

If MC >AC then extra units add more than proportionately to costs, so AC is rising
Similarly, if MC < AC, AC is falling

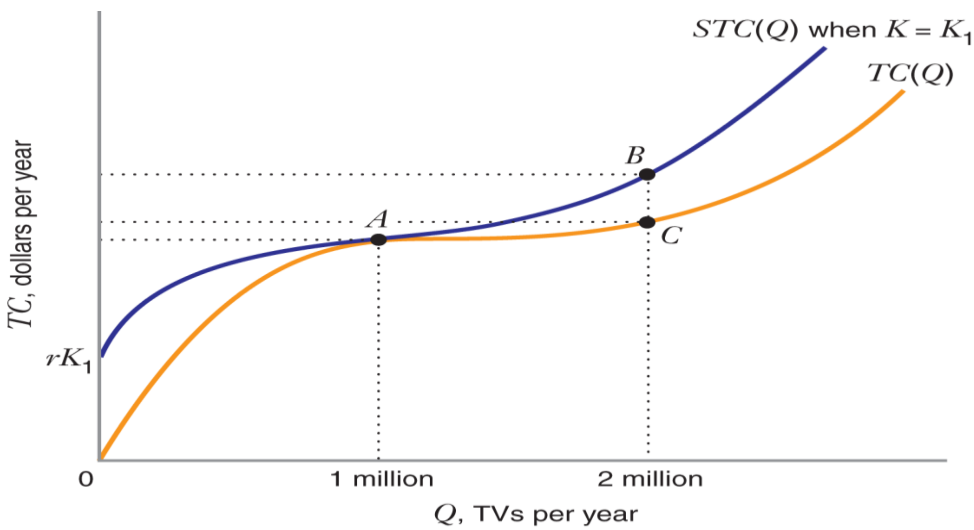
Deriving a Short-Run Total Cost Curve from Isoquants and Isocost Curves

In the diagram below, initially, the firm wants to produce 1 million television sets per year. In the long run, when it is free to vary both capital and labor, it minimizes total cost by operating at point A, using L_1 units of labor and K_1 units of capital.

Suppose the firm wants to increase its output to 2 million TVs per year and that, in the short run, its usage of capital must remain fixed at K_1. In that case, the firm would operate at point B, using L_3 units of labor and the same K_1 units of capital. In the long run, however, the firm could move along the expansion path and operate at point C, using L_2 units of labor and the same K_2 units of capital. Since point B is on a higher isocost line than point C, the short-run total cost is higher than the long-run total cost when the firm is producing 2 million TVs per year.

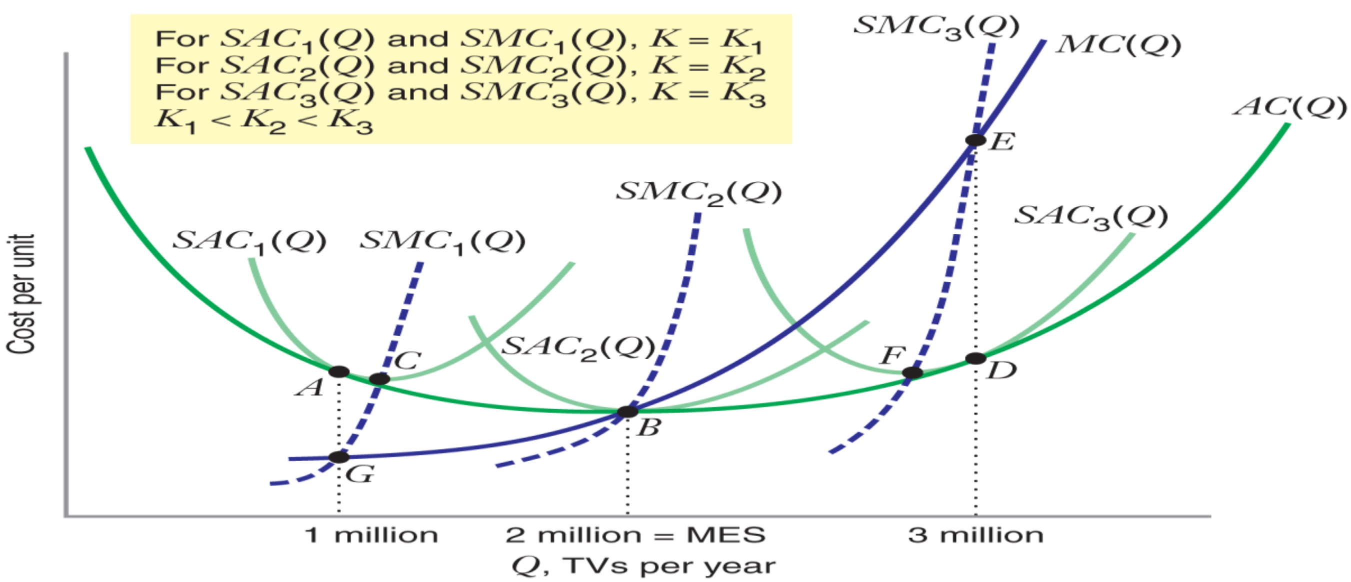


The firm's corresponding long-run and short-run total cost curves are TC(Q) and STC(Q) shown in the diagram below. We see that STC(Q) always lies above TC(Q) (i.e., short-run total cost is greater than long-run total cost) except at point A, where STC(Q) and TC(Q) are equal.



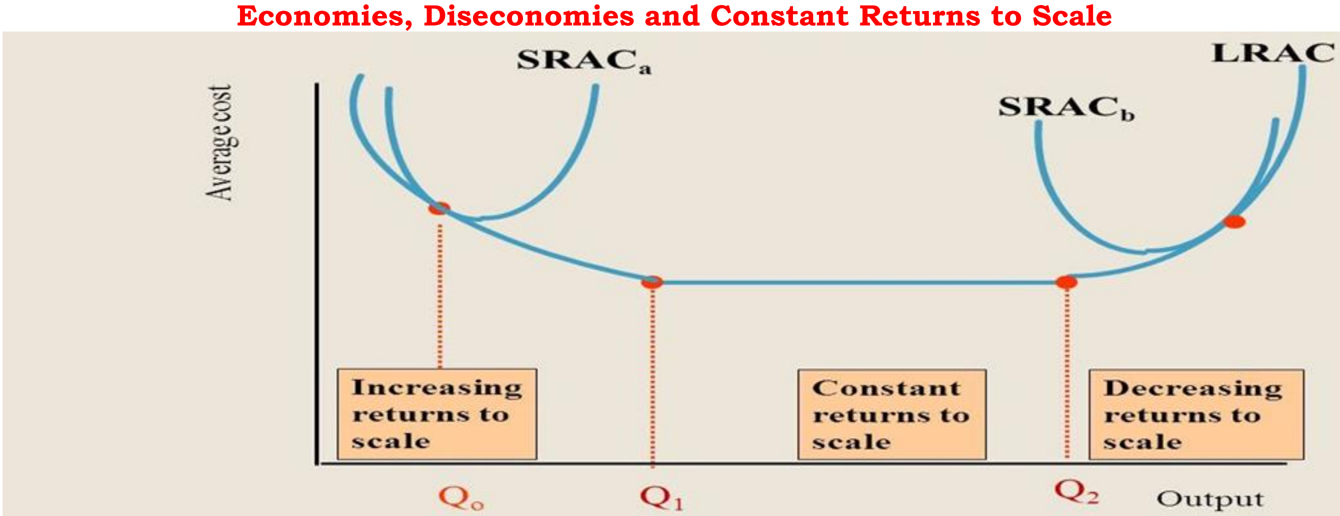
The Long-Run Average Cost Curve as an Envelope Curve

The long-run average cost curve forms a boundary (or envelope) around the set of short-run average cost curves corresponding to different levels of output and fixed input. Figure 8.17 illustrates this for a producer of television sets. The firm's long-run average cost curve AC(Q) is U-shaped, as are its short-run average cost curves SAC1(Q), SAC2(Q), and SAC3(Q), which correspond to different levels of fixed capital K_1, K_2, and K_3 (where $K_1 < K_2 < K_3$).



In Chapter 7, we discussed the concept of Economies, Diseconomies and Constant Returns to Scale. Diagrammatically this is shown below.

Economies, Diseconomies and Constant Returns to Scale



The minimum point on the LRAC curve is we call the **optimal size or output of the firm**.

Example 1:

A firm produces a product with labor and capital, and its production function is described by Q = $Q = K^{\frac{1}{2}} L^{\frac{1}{2}}$ K. Suppose that the price of labor and the price of capital equals 1. Derive the equations for the long-run total cost curve and the long-run average cost curve.

We see that the = $MP_K = \frac{1}{2} K^{-\frac{1}{2}} L^{\frac{1}{2}}$ and $MP_L = \frac{1}{2} K^{\frac{1}{2}} L^{-\frac{1}{2}}$ and based on our knowledge of production theory the firm will hire input such that. $\frac{MP_L}{MP_K} = \frac{w}{r}$ *or* $\frac{\frac{1}{2} K^{\frac{1}{2}} L^{-\frac{1}{2}}}{\frac{1}{2} L^{\frac{1}{2}} K^{-\frac{1}{2}}} = \frac{1}{1}$ *or* $K = L$.

TC = wL + rK or TC = L + L or TC = 2L Substituting K=L into the production function yields $Q = L^{\frac{1}{2}} L^{\frac{1}{2}} = L$. We can then solve for L in terms of Q. $L = Q$

Plugging back into the cost function yields $TC = 2Q$ $MC = \frac{dTC}{dQ} = 2$ $AC = \frac{2Q}{Q} = 2$

We can see the long-run average cost function is horizontal. Constant Returns to Scale.

Example 2: Consider the following Total Cost curve equation:

Assume this is the total cost curve of a firm: TC = 145Q - 18Q^2 + 3Q^3. What is the optimal output of the firm?

To answer that question, we need to know the average cost curve and the minimum point on that curve. This can be found two ways. We know two things about the ATC curve at the optimal point.

One, which is where the MC curve intersects the ATC.

Two, we know the slope of the ATC curve is zero at that point.

First, what is the equation for the ATC curve: $ATC = \frac{TC}{Q} = 145 - 18Q + 3Q^2$

The minimum point is where the ATC curve has a slope of zero or where the derivative with respect to Q is equal to zero (you can assume the appropriate second order conditions hold). Therefore,

$\frac{dATC}{dQ} = -18 + 6Q$ Set that to zero and solve for Q:

$$6Q - 18 = 0$$
$$6Q = 18$$
$$Q = 3$$

We can check our result by finding the MC curve, setting that equal to ATC curve, and solving for Q. Marginal cost is the derivation of the TC curve with respect to Q, that gives us the following equation:

$$MC = \frac{dTC}{dQ} = 145 - 36Q + 9Q^2 = 145 - 18Q + 3Q^2 \quad = 18Q - 6Q^2 = 0$$

Solving for Q gives Q=3, confirming our original result. Therefore, the optimal output of the firm is 3 units. At 3 units, ATC = $118.

ADDITIONAL TOPICS

Economies of Scope

We have concentrated on cost curves for firms that produce just one product or service. In reality, though, many firms produce more than one product. For a firm that produces two products, total costs would depend on the quantity Q_1 of the first product the firm makes and the quantity Q_2 of the second product it makes. We will use the expression $TC(Q_1, Q_2)$ to denote how the firm's costs vary with Q_1 and Q_2.

In some situations, efficiencies arise when a firm produces more than one product. That is, a two-product firm may be able to manufacture and market its products at a lower total cost than two single-product firms. These efficiencies are called economies of scope.

Economies of Experience: The Experience Curve

Economies of scale refer to the cost advantages that flow from producing a larger output at a given point in time. Economies of experience refer to cost advantages that result from accumulated experience over an extended period of time, or from learning-by-doing, as it is sometimes called.

The magnitude of cost reductions that are achieved through experience is often expressed in terms of the slope of the experience curve, which tells us how much average variable costs go down as a percentage of an initial level when cumulative output doubles.

For example, if doubling a firm's cumulative output of semiconductors results in average variable cost falling from $10 per megabyte to $8.50 per megabyte, we would say that the slope of the experience curve for semiconductors is 85 percent, since average variable costs fell to 85 percent of their initial level.

In terms of an equation, slope of experience curve=AVC(2N)/AVC(N)