CHAPTER 7 LECTURE – COSTS AND COST MINIMIZATION

The ***opportunity cost*** of an asset (or, more generally, of a choice) is the highest valued opportunity that must be passed up to allow current use. Thus, the monthly opportunity cost of a motorcycle owned by Ms. Hien (motorcycle taxi driver) may be, for example, the monthly income the motorcycle could have generated if Ms. Hien had rented it for someone else to use.

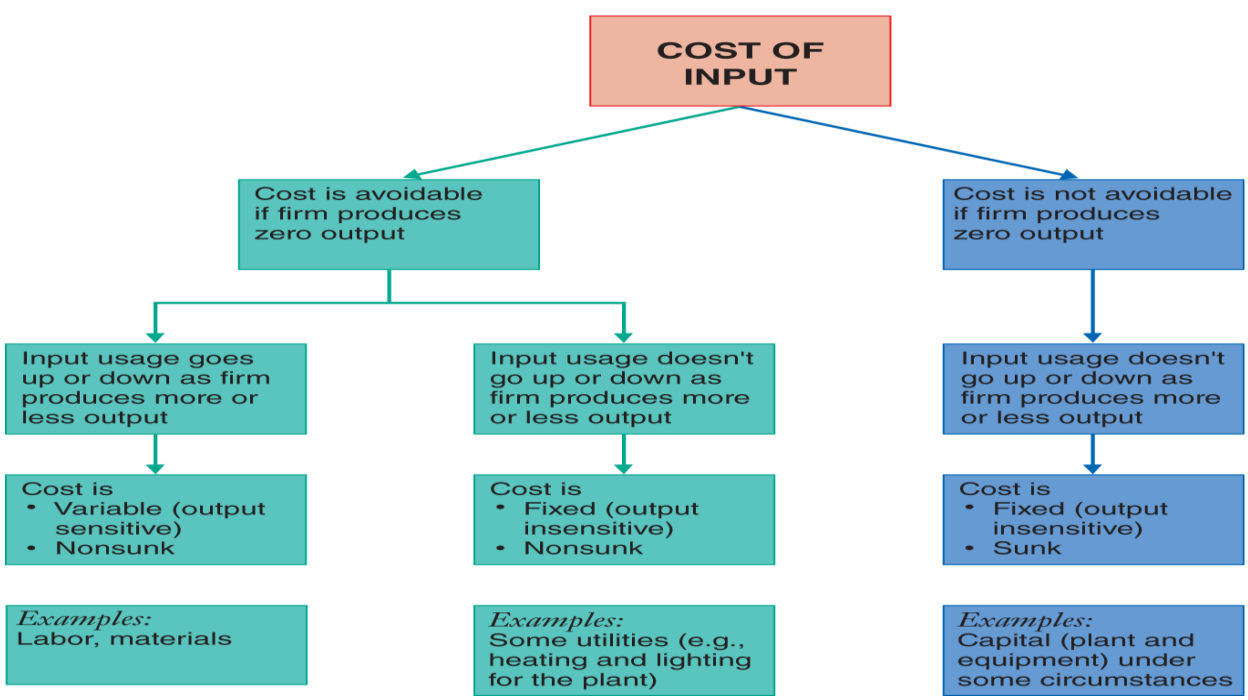
Explicit costs are expenses for which one must pay with cash or equivalent. Because a cash transaction is involved, they are relatively easily accounted for in analysis.

Implicit costs do not involve a cash transaction, and so we use the opportunity cost concept to measure them. This analysis requires detailed knowledge of alternatives that were not selected at various decision points. Relevant here are the opportunity cost of the firm's assets and cash, and of the owner's time invested in the firm.

Incremental cost is the change in cost caused by a particular managerial decision. Thus the increment is at the decision level, and may involve multiple units of change in output or input. Incremental costs may be involved when considering a product or service modification or a change in production process.

Sunk costs are those parts of the purchase cost that cannot later be salvaged or modified through resale or other changes in operations. Image advertising for a new product is a classic example of a sunk cost, as is an option or investment in assets whose value is specific to a particular situation. Sunk costs reflect *commitment*, or irreversibility, and so are not a part of incremental analysis.

Fixed cost or Sunk Cost



Accounting costs: measure **historical** costs, or costs actually paid.

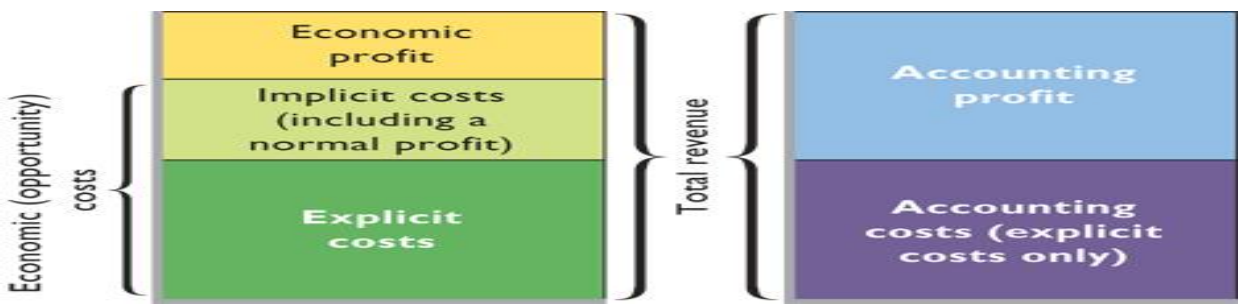
Economic costs measure **opportunity costs**, or the cost in terms of the best forgone alternative.

Normal Profit - minimum level of profit required to keep the factors of production in their current use in the long run.

Also looked at as the minimum profit necessary to attract and retain suppliers in a perfectly competitive market). Only a normal profit could be earned in such markets because, if profit was abnormally high, more competitors would appear and drive prices and profit down. If profit was abnormally low, firms would leave the market and the remaining ones would drive the prices and profit up. Markets where suppliers are making normal profits will neither expand nor shrink and will, therefore, be in a state of long-term equilibrium. Normal profit typically equals opportunity cost.

Economic profit arises when its revenue exceeds the total (opportunity) cost of its inputs, noting that these costs include the cost of equity capital that is met by "normal profits."

A business is said to be making an **accounting profit** if its revenues exceed the accounting cost the firm "pays" for those inputs. Economics treats the normal profit as a cost, so when deducted from total accounting profit what is left is economic profit (or economic loss).



Short-Run and Long-Run Costs

In microeconomics and managerial economics, the ***short run*** is the decision-making period during which at least one input is considered fixed. The fixed input is commonly considered to be some aspect of capital, such the production facility, but may also be a normally variable input that is fixed because of production technology requirements, or a contractual commitment (e.g., a facility lease) related to production. So when one refers to short-run analysis, the analysis is focused on a planning period in which some input is fixed and others are variable, and the manager is selecting levels of variable input and production output to optimize given the constraint of the fixed input.

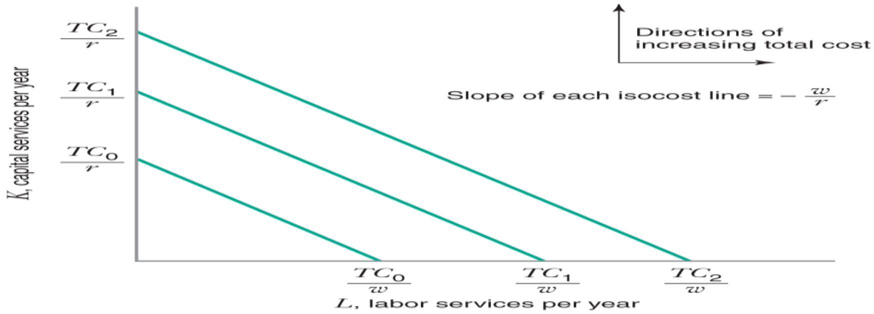
In contrast, the economic ***long run*** is a planning horizon that looks beyond current commitments to a future period in which all inputs can be varied. A typical long-run analytical problem is the decision of whether to adjust capacity, seek a larger (or smaller) facility, to change product lines, or to adopt a new technology.

The Long-Run

Isocosts - firms have an isocost that allows them to consider various combinations of inputs that yield the same total cost. As in the demand analysis unit earlier in the semester, we start with a budget equation: $C = wL + rK$

Where *w* and *r* represent the Marginal factor cost (MFC) of L and K (assuming a competitive factor market – which means MFC is given) we can derive an equation of the line:

$K = \frac{C}{r} - \frac{w}{r}L$. The slope of the isocost is $-\frac{w}{r}$, which says that the price ratio of the inputs is equal to the slope of the isocost line.



The (Long Run) Cost Minimisation Problem

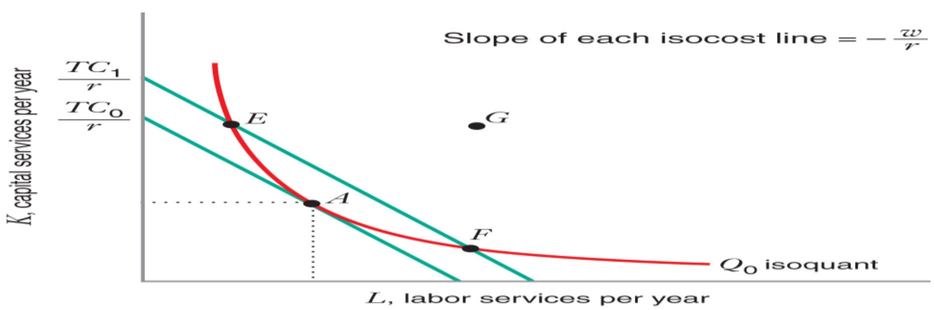
Suppose that a firm's owners wish to minimise costs of a given output (this is the same result of maximizing output for a given level of costs – the Duality concept)

Let the desired output be Q*

Q = Q(K,L) Owner's problem: min TC = rK + wL Subject to Q* = Q(K, L)

Cost minimization subject to satisfaction of the isoquant equation: Q* = f(K, L)

Tangency condition: $MRTS_{L,K} = -MP_L/MP_K = -w/r$ or $MP_L/MP_K = w/r$



The cost-minimizing input combination occurs where:

$\frac{MP_L}{MP_K} = \frac{w}{r}$ This can also be written as $\frac{MP_L}{w} = \frac{MP_K}{r}$

More Formal Mathematics of Cost-Minimization/Profit Maximization

Cost minimizing input choices. The above problem can be viewed as a profit maximization problem (where we select quantity). We show the analysis as a cost minimization problem, where K and L are selected, for a given output level, Q*.

This is a constrained minimization problem, hence, the Lagrangean function is:

$$£ = wL + rK + \lambda[(Q^* - f(K,L)]]$$

Taking the derivative of £ with respect to each good (i.e., *X* and *Y*) and setting it equal to zero:

$$\frac{\partial £}{\partial L} = w - \lambda \frac{\partial f}{\partial L} = 0 \qquad \frac{\partial £}{\partial K} = r - \lambda \frac{\partial f}{\partial K} = 0$$

Solving for λ: $\lambda = \frac{MP_L}{w}$ and $\lambda = \frac{MP_K}{r}$

Equating these two equations yields the same result as the before: $\frac{MP_L}{w} = \frac{MP_K}{r}$

We can look at the meaning of the Lagrange multiplier λ. The value of λ equals the additional input cost per additional unit of output generated by an input, namely, λ=ΔTCΔQ. That is, the Lagrange multiplier measures the firm's incremental cost from increasing its output by a small amount.

Example:

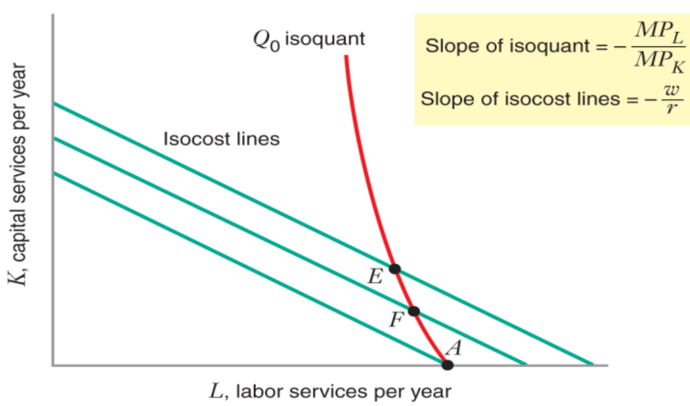
$Q = 50L^{1/2}K^{1/2}$ $MP_L = 25L^{-1/2}K^{1/2}$ $MP_K = 25L^{1/2}K^{-1/2}$

w = \$5 r = \$20 Q* = 1000 $MP_L/MP_K = K/L$

K/L = 5/20…or…L=4K Putting this in the Isocost function.

$50L^{1/2}K^{1/2} = 1000$ $50(4K)^{1/2}K^{1/2} = 100K = 1000$ K = 10; L = 40

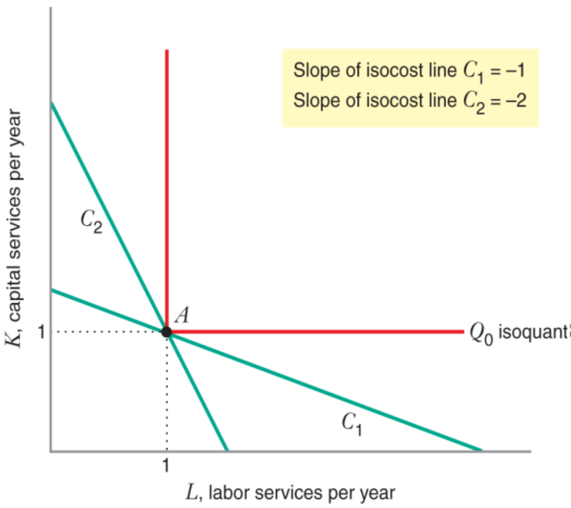
Corner Point Solution to the Cost-Minimization Problem: The cost-minimizing input combination occurs at point A, where the firm uses no capital. Points such as E and F cannot be cost minimizing, because the firm can lower costs and keep output the same by substituting labor for capital.

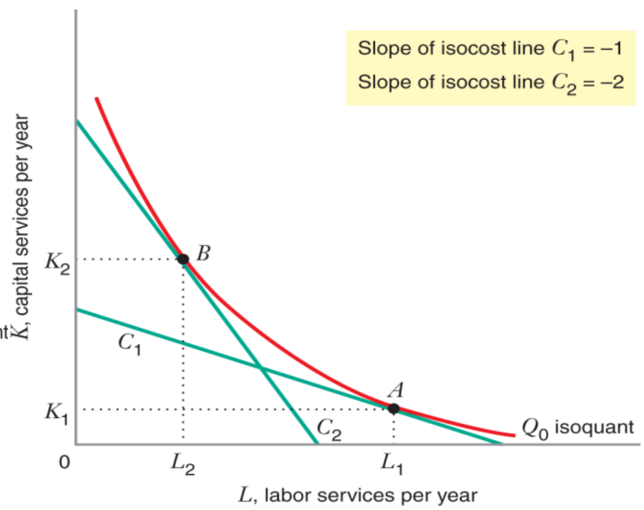


COMPARATIVE STATICS ANALYSIS OF THE COST-MINIMIZATION PROBLEM

Comparative Statics Analysis of Cost-Minimization Problem with Respect to the Price of Labor: *In the graph at the right*, the price of capital r = 1 and the quantity of output Q_0 are held constant. When the price of labor is w = 1, the isocost line is C1 and the ideal input combination is at point A (L1, K1). When the price of labor is w = 2, the isocost line is C2 and the ideal input combination is at point B (L2, K2). Increasing the price of labor causes the firm to substitute capital for labor.

In the graph on the left, there is a **Fixed-Proportions Production Function** and change in the input price ratio does not change the production decision.





EXAMPLES

1). Suppose the production of airframes is characterized by a Cobb–Douglas production function: *Q* = *LK*. Suppose the price of labor is $10 per unit and the price of capital is $1 per unit. Find the cost-minimizing combination of labor and capital if the manufacturer wants to produce 121,000 airframes.

You can prove the marginal products for this production function are

$MP_L = K$ and $MP_K = L$.

The tangency condition implies $10 = \dfrac{K}{L}$ *or* $10L = K$

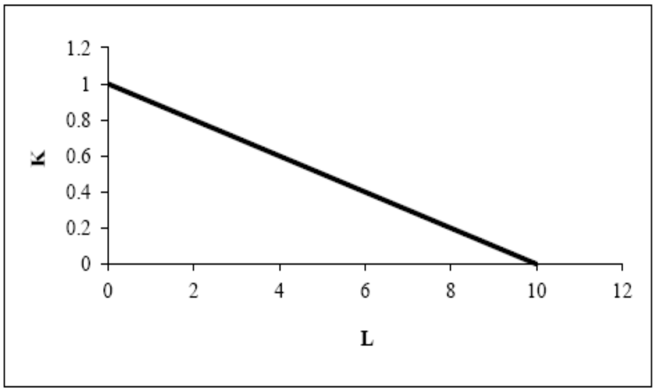
Substituting into the production function yields

$121{,}000 = LK \quad 121{,}000 = L(10L) \quad 121{,}000 = 10L^2 \quad 12{,}100 = L^2 \quad L = 110$

Since $K = 10L$, $K = 1{,}100$. The cost-minimizing quantities of labor and capital to produce 121,000 airframes are $K = 1{,}100$ and $L = 110$.

2). The processing of payroll for the 10,000 workers in a large firm can either be done using 1 hour of computer time (denoted by *K*) and no clerks or with 10 hours of clerical time (denoted by *L*) and no computer time. Computers and clerks are perfect substitutes; for example, the firm could also process its payroll using 1/2 hour of computer time and 5 hours of clerical time.

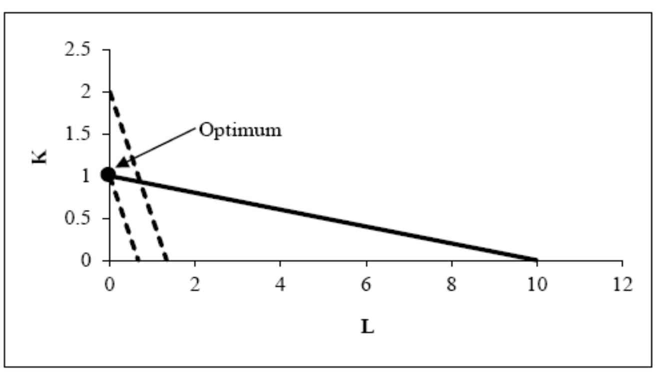
a) Sketch the isoquant that shows all combinations of clerical time and computer time that allows the firm to process the payroll for 10,000 workers.



K and L are perfect substitutes, meaning that the production function is linear and the isoquants are straight lines. We can write the production function as $Q = 10{,}000K + 1000L$, where Q is the number of workers for whom payroll is processed.

b) Suppose computer time costs $5 per hour and clerical time costs $7.50 per hour. What are the cost-minimizing choices of *L* and *K*? What is the minimized total cost of processing the payroll?

If $r = 5$ and $w = 7.50$, the slope of a typical isocost line will be $-7.5/5.0 = -1.5$. This is steeper than the isoquant implying that the firm will employ only computer time (K) to minimize cost. The cost minimizing combination is $K = 1$ and $L = 0$. This outcome can be seen in the graph below. The isocost lines are the dashed lines.



The total cost to process the payroll for 10,000 workers will be $TC = 5(1) + 7.5(0) = 5$.

c) Suppose the price of clerical time remains at $7.50 per hour. How high would the price of an hour of computer time have to be before the firm would find it worthwhile to use only clerks to process the payroll?

The firm will employ clerical time only if MP_L / w > MP_K / r. Thus we need 0.1 / 7.5 > 1/r or r > 75.

3). A firm operates with the production function Q = K²L. Q is the number of units of output per day when the firm rents K units of capital and employs L workers each day. You can see that the marginal product of capital is 2KL, and the marginal product of labor is K². The manager has been given a production target: Produce 8,000 units per day. She knows that the daily rental price of capital is $400 per unit. The wage rate paid to each worker is $200 day.

a) Currently the firm employs at 80 workers per day. What is the firm's daily total cost if it rents just enough capital to produce at its target?

Suppose that the firm is operating in the short run, with L = 80. To produce Q = 8,000, how much K will it require? From the production function we observe that 8,000 = K^2 (80) and we can see K = 10.

The total cost would be C = wL + rK = $200(80) + $400(10) = $20,000 per day.

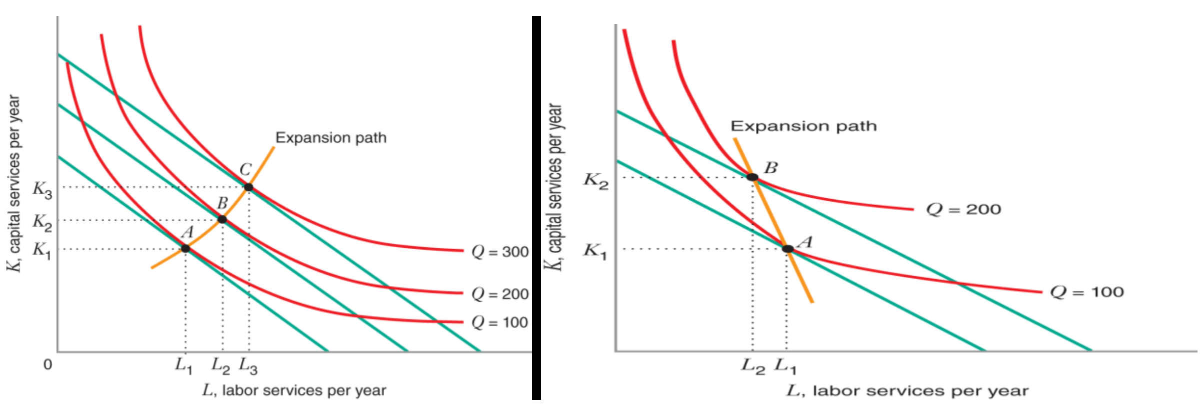
b) Compare the marginal product per dollar sent on K and on L when the firm operates at the input choice in part (a). What does this suggest about the way the firm might change its choice of K and L if it wants to reduce the total cost in meeting its target?

Let's examine the "bang for the buck" for K and L when K = 10 and L = 80.

For capital: MP_K / r = 2KL / 400 = 2(10)(80) / 400 = 4
For labor: MP_L / w = K^2 / 200 = 10^2 / 200 = 0.5
So, the marginal product per dollar spent on capital exceeds that of labor. The firm would like to rent more capital and hire fewer workers.

COMPARATIVE STATICS ANALYSIS OF CHANGES IN OUTPUT

Both Inputs Normal Inputs **Labor is an Inferior Input**



DERIVING THE INPUT DEMAND CURVES

Consider the production function Q = LK. Suppose that the price of labor equals w and the price of capital equals r. Derive expressions for the input demand curves.

We can see the marginal products are MPL = K and MPK = L.

From the tangency condition, we get $K/L = w/r \Rightarrow K = (w/r)L$

Substituting into the production function Q = LK yields

$Q = L\left(\frac{w}{r}\right)L \qquad Q = \left(\frac{w}{r}\right)L^2 \qquad L = \left(\frac{rQ}{w}\right)^{1/2}$

This represents the input demand curve for L. Since $K = \left(\frac{w}{r}\right)L$ we have

$K = \left(\frac{w}{r}\right)\left(\frac{rQ}{w}\right)^{1/2} \qquad K = \left(\frac{wQ}{r}\right)^{1/2}$

This represents the input demand curve for K.

The Demand for Labor

Suppose that the firm's production function is given by the production function

$Q = 50K^{1/2}L^{1/2}$.

The firm's capital is fixed at $\bar{K}$. What amount of labor will the firm hire to minimize cost in the short run?

Solution: Since output is given as Q and capital is fixed at $\bar{K}$, the equation for the production function contains only one unknown, L:

$Q = 50\bar{K}^{1/2}L^{1/2}$

Solving this equation for L gives us $L = \frac{Q^2}{2500\bar{K}}$

THE PRICE ELASTICITY OF DEMAND FOR INPUTS

Elasticity of the input (L) is defined as $E_{L,w} = \frac{\partial L}{\partial w}\frac{w}{L}$

Elasticity of the input (K) is defined as $E_{K,r} = \frac{\partial K}{\partial r}\frac{r}{K}$

The Price Elasticity of Demand for Labor Depends on the Elasticity of Substitution Between Labor and Capital

Referring to the figure below, the price of labor decreases from $2 to $1, with the price of capital and quantity of output held constant.

In panels (a) and (b), the elasticity of substitution is low (0.25), so the 50 percent decrease in the price of labor results in only an 8 percent increase in the quantity of labor (i.e., demand for labor is relatively insensitive to price of labor; the cost-minimizing input combination moves only from point A to point B).

In panels (c) and (d), the elasticity of substitution is high (2), so the same 50 percent decrease in the price of labor results in a 127 percent increase in the quantity of labor (i.e., demand for labor is much more sensitive to price of labor; the movement of the cost-minimizing input combination from point A to point B is much greater).