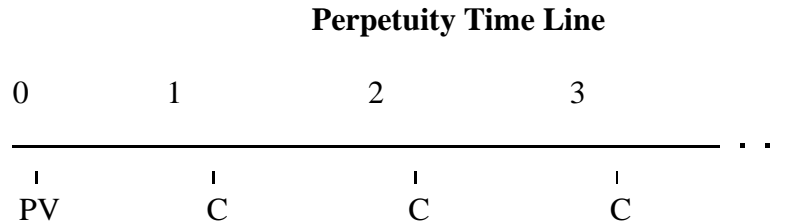


Perpetuities

A **perpetuity** is a series of equal payments over an infinite time period into the future. Consider the case of a cash payment C made at the end of each year at interest rate i , as shown in the following time line:



Because this cash flow continues forever, the present value is given by an infinite series:

$$PV = C/(1+i) + C/(1+i)^2 + C/(1+i)^3 + \dots$$

From this infinite series, a usable present value formula can be derived by first dividing each side by $(1+i)$.

$$PV/(1+r) = C/(1+r)^2 + C/(1+r)^3 + C/(1+r)^4 + \dots$$

In order to eliminate most of the terms in the series, subtract the second equation from the first equation:

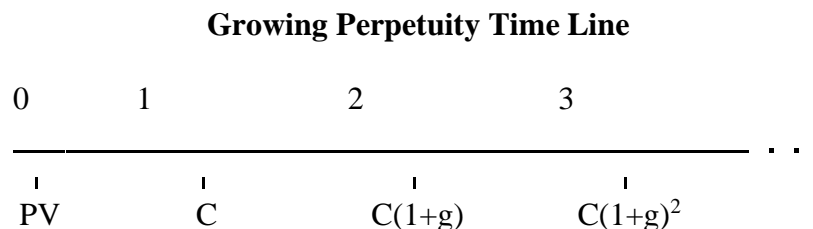
$$PV - PV/(1+r) = C/(1+r)$$

Solving for PV , the present value of a perpetuity is given by:

$$PV = \frac{C}{r}$$

Growing Perpetuities

Sometimes the payments in a perpetuity are not constant but rather, increase at a certain growth rate g as depicted in the following time line:



The present value of a growing perpetuity can be written as the following infinite series:

$$PV = \frac{C}{(1+r)} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \dots$$

To simplify this expression, first multiply each side by $(1+g)/(1+r)$:

$$\frac{PV(1+g)}{(1+r)} = \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \dots$$

Then subtract the second equation from the first:

$$PV - \frac{PV(1+g)}{(1+r)} = \frac{C}{(1+r)}$$

Finally, solving for PV yields the expression for the present value of a growing perpetuity:

$$PV = \frac{C}{r-g}$$

For this expression to be valid, the growth rate must be less than the interest rate, that is, $g < r$.

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