

## Arithmetic Mean versus Geometric Mean

Adapted from <https://www.educba.com/geometric-mean-vs-arithmetic-mean/>  
<https://www.wallstreetmojo.com/geometric-mean-vs-arithmetic-mean/>

The Arithmetic mean and Geometric mean are the tools widely used to calculate growth rates or the returns on investment for investment portfolios in the world of finance. People too often use the arithmetic mean to report the higher returns which are not the correct measure of calculating the return on investment or growth. This is because growth rates and return on investment for a portfolio over the years is dependent on returns in previous years. Thus, the Geometric mean is the correct way to calculate the return on investment for a specific time period. The arithmetic mean is better suited in the situation wherein variables being used for the calculation of average are not dependent on each other.

### Example: Suitability use of Geometric mean vs Arithmetic mean

1. Let's take an example of return on investment for an amount of \$100 over 2 years. Suppose the returns in two years were -50% and +50% in the 1<sup>st</sup> and 2<sup>nd</sup> Average return calculation by using arithmetic mean will be 0% (**Arithmetic mean** =  $(-50\%+50\%) / 2 = 0\%$ )

Which gives a wrong impression that the investor is breaking even on its investment, and there is no loss or profit. However, a closer analysis gives the entirely different picture of the scenario.

Year	1	2
Starting Investment (a)	100	50.00
Return (%)	-50.00%	50.00%
Gain/(Loss) in the Year (b)=(a*return%)	-50.00	25.00
Closing Balance (a+b)	50.00	75.00

From the above table, we can see that the investment of \$100 after -50% and +50% return in year 1 and 2, will be close to \$75. Therefore, the investor is not breaking even on its investment as suggested by the arithmetic mean average, but he has incurred a loss of \$25 after 2 years on its investment. This is well reflected by using Geometric mean to calculate the return on the investment over 2 years as below:

### The geometric mean of returns

This means the annualized return on the portfolio had been **negative** at 13.40%. The investment position after two years is as below:

Year	1	2
Starting Investment (a)	100	86.60
Return (%)	-13.40%	-13.40%
Gain/(Loss) in the Year (b)=(a*return%)	-13.40	-11.60
Closing Balance (a+b)	86.60	75.00

Therefore, the Geometric mean shows the true picture of investment that there is a loss in investment with an annualized negative return of -13.40%. Since the return in each year impacts the absolute return in the next year, a geometric mean is a better way to calculate the annualized return on investment.

2. When one needs to calculate the average of variables that are not dependent on each other, Arithmetic means a suitable tool to calculate the average. The average of marks of a student for 5 subjects can be calculated by the arithmetic mean as scores of the student in different subjects are independent of each other.

## How to Calculate the Geometric Average

The geometric mean for a series of numbers is calculated by taking the product of these numbers and raising it to the inverse of the length of the series.

To do this, we add one to each number (to avoid any problems with negative percentages). Then, multiply all the numbers together and raise their product to the power of one divided by the count of the numbers in the series. Then, we subtract one from the result.

$$\text{Geometric Mean} = \left[ (1 + r_1) \times (1 + r_2) \times \dots \times (1 + r_n) \right]^{\frac{1}{n}} - 1$$

Using the information above.

$$R_1 = -50\% \quad R_2 = 50\%$$

$$\text{Geometric mean} = [(0.50) \times (1.50)]^{1/2} - 1 = (0.75)^{1/2} - 1 = 0.86 - 1 = -13.4\%$$