

Choice under Uncertainty Problems – Chapter 6

1. Your bike is worth \$16. There is a 50% chance that it will be stolen from the dining hall at lunch today. Your utility function for the bike is $U = (\text{bike value})^2$
 - a. Are you risk averse, a risk lover, or risk neutral?
 - b. What is the expected value of your bike considering its vulnerability?
 - c. The campus security has a bike check-in that will guard your bike for \$5, so there would be no risk of loss. Do you take the campus security deal?
 - d. If your answer is yes, explain why. If your answer is no, what is the maximum you would pay security to check in your bike?
 - e. Next, suppose your bike utility function is $U = 15(\text{bike value})$. Are you risk averse, a risk lover, or risk neutral?

Solution

- a) Risk lover
- b) $EV = 0.5(16) + 0.5(0) = \8
- c) No, you get 128 utility from risk taking and only 121 from a sure \$11.

$$EU(\text{Bike}) = 0.5(16^2) = 128 \quad U = 11^2 = 121$$

- d) The bike value squared must equal 128 units of pleasure. Since the square root of 128 is 11.31, you will be willing to pay up to 4.69.
- e) Risk neutral

2. Suppose you have \$10,000 to invest. A broker phones you with some information you requested on certain junk bonds. If the company issuing the bonds posts a profit this year, it will pay you a 40 per cent interest rate on the bond. If the company files for bankruptcy, you will lose all you invested. If the company breaks even, you will earn a 10 per cent interest rate. Your broker tells you there is a 50 per cent chance that they will break even and a 20 per cent chance that the company will file for bankruptcy. Your other option is to invest in a risk-free government bond that will guarantee 8 per cent interest for 1 year.

- a. What is the expected interest rate for the junk bond investment?
- b. Which investment will you choose if your utility function is given by $U = M^2$?
- c. Which investment will you choose if your utility function is given by $U = \sqrt{M}$.

Solution

- a) Possible outcomes are 40% with probability 0.3, -100% with probability 0.2, and 10% with probability 0.5

$$E(\text{interest rate}) = 0.3(0.4) + 0.2(-1) + 0.5(0.1) = -0.03 = -3\%$$

b) $EU(\text{govt. bond}) = (1.08 \times 10,000)^2 = 116,640,000$

$$EU(\text{junk bond}) = 0.3(1.4 \times 10,000)^2 + 0.2(0)^2 + 0.5(1.1 \times 10,000)^2 = 119,300,000$$

Since $EU(\text{junk bond}) > EU(\text{govt. bond})$, you will invest in the junk bond.

c) $EU(\text{govt. bond}) = \sqrt{1.08 \times 10,000} = 103.92$

$$EU(\text{junk bond}) = 0.3\sqrt{1.4 \times 10,000} + 0.2\sqrt{0} + 0.5\sqrt{1.1 \times 10,000} = 87.94 .$$

Since $EU(\text{junk bond}) < EU(\text{govt. bond})$, you will invest in the govt. bond.