INTERMEDIATE MICROECONOMICS LECTURE 9 – THE COSTS OF PRODUCTION

The ***opportunity cost*** of an asset (or, more generally, of a choice) is the highest valued opportunity that must be passed up to allow current use. Thus, the monthly opportunity cost of a motorcycle owned by Ms. Hien (motorcycle taxi driver) may be, for example, the monthly income the motorcycle could have generated if Ms. Hien had rented it for someone else to use.

Explicit costs are expenses for which one must pay with cash or equivalent. Because a cash transaction is involved, they are relatively easily accounted for in analysis.

Implicit costs do not involve a cash transaction, and so we use the opportunity cost concept to measure them. This analysis requires detailed knowledge of alternatives that were not selected at various decision points. Relevant here are the opportunity cost of the firm's assets and cash, and of the owner's time invested in the firm.

Incremental cost is the change in cost caused by a particular managerial decision. Thus the increment is at the decision level, and may involve multiple units of change in output or input. Incremental costs may be involved when considering a product or service modification or a change in production process.

Sunk costs are those parts of the purchase cost that cannot later be salvaged or modified through resale or other changes in operations. Image advertising for a new product is a classic example of a sunk cost, as is an option or investment in assets whose value is specific to a particular situation. Sunk costs reflect *commitment*, or irreversibility, and so are not a part of incremental analysis.

Accounting costs: measure **historical** costs, or costs actually paid.

Economic costs measure **opportunity costs**, or the cost in terms of the best forgone alternative.

Accounting costs differ mainly in terms of two types of goods: Durable goods and inputs not directly purchased.

The historical cost of a building is measured using some conventional depreciation rate. For example, if we buy a building for $10,000,000, accounting practices generally assume a 5% depreciation rate (a life span of 20 years). Then we allocate the $10,000,000 to annual production costs at the rate of $500,000 the first year, $475,000 the second year, and so on. If the firm borrows to buy the building, the interest paid each year is also an historical cost incurred in production.

The economic cost of the building is established by calculating the implicit rental rate: this is the rent that the firm implicitly pays to itself for the use of the durable inputs that it owns. We calculate it as an opportunity cost: the firm could have made money renting the building out to some other firm. Also, instead of depreciation according to some formula, economists calculate depreciation in terms of the change in the market value of the durable input. The opportunity cost is what was lost by not selling the building at the beginning of the year.

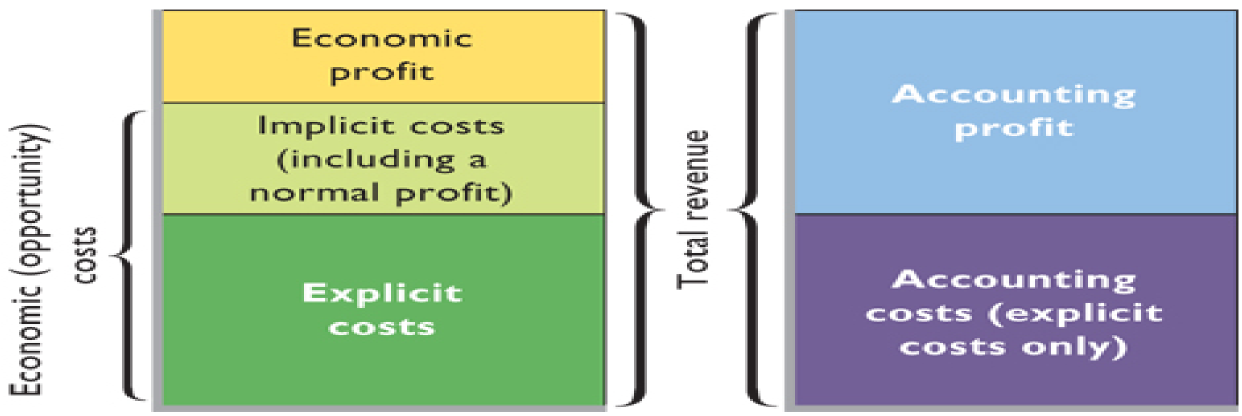
This kind of thinking also applies to owner's wages: the opportunity cost to the entrepreneur is the wage he could have made working for someone else.

Normal Profit - minimum level of profit required to keep the factors of production in their current use in the long run.

Also looked at as the minimum profit necessary to attract and retain suppliers in a perfectly competitive market). Only a normal profit could be earned in such markets because, if profit was abnormally high, more competitors would appear and drive prices and profit down. If profit was abnormally low, firms would leave the market and the remaining ones would drive the prices and profit up. Markets where suppliers are making normal profits will neither expand nor shrink and will, therefore, be in a state of long-term equilibrium. Normal profit typically equals opportunity cost.

Economic profit arises when its revenue exceeds the total (opportunity) cost of its inputs, noting that these costs include the cost of equity capital that is met by "normal profits."

A business is said to be making an **accounting profit** if its revenues exceed the accounting cost the firm "pays" for those inputs. Economics treats the normal profit as a cost, so when deducted from total accounting profit what is left is economic profit (or economic loss).



Short-Run and Long-Run Costs

In microeconomics and managerial economics, the ***short run*** is the decision-making period during which at least one input is considered fixed. The fixed input is commonly considered to be some aspect of capital, such the production facility, but may also be a normally variable input that is fixed because of production technology requirements, or a contractual commitment (e.g., a facility lease) related to production. So when one refers to short-run analysis, the analysis is focused on a planning period in which some input is fixed and others are variable, and the manager is selecting levels of variable input and production output to optimize given the constraint of the fixed input.

In contrast, the economic ***long run*** is a planning horizon that looks beyond current commitments to a future period in which all inputs can be varied. A typical long-run analytical problem is the decision of whether to adjust capacity, seek a larger (or smaller) facility, to change product lines, or to adopt a new technology.

Short Run Cost Curves

In order to examine a firm's costs, we need to know its optimal input mix and the prices of the inputs. Then we can define the important cost concepts:

Total Cost = TC Total Fixed Cost = TFC
Total Variable Cost = TVC Average Total Cost = ATC = TC/Q
Average Variable Cost = ATC = TVC/Q
Average Fixed Cost = TFC/Q

Marginal Cost = $\Delta TC/\Delta Q = \Delta TVC/\Delta Q = \frac{dTC}{dQ} = \frac{dTVC}{dQ}$

Total Cost is made up of two components in the short run: Total Fixed Cost and Total Variable Cost: TFC + TVC = TC.

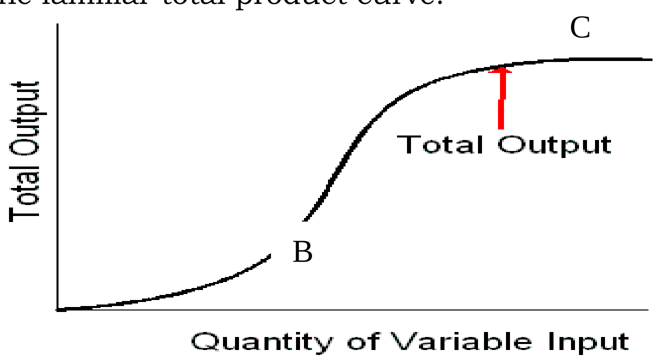
TFC does not vary with the output rate. TFC is the cost of the fixed inputs. In the simple case that we examined where capital (K) was the input fixed in the short run.

NOTE: We will use *w* to represent the price of labor and *r* to represent the cost of capital. Some books use P_L and P_K .

TVC does vary with output rate. TVC is the cost of the variable inputs. In the simple case that we examined where labor (L) was the input that varied in the short run.

TVC = wL TC = TFC + TVC

In the short run, then, we have some fixed inputs and some variable inputs that yield the familiar total product curve:



A **From AB let production function be $Q=L^2$**

$dQ/dL = MP_L = 2L$.
$d^2Q/dL^2 = 2 > 0$
Increasing marginal returns
TVC = $wQ^{½} = Q^{½}$ Let w = 1

$dTVC/dQ = ½Q^{-½}$
$d^2TVC/dQ^2 = -1/4Q^{-3/2} < 0$

Marginal Cost is downward sloping. TVC rises at decreasing rate

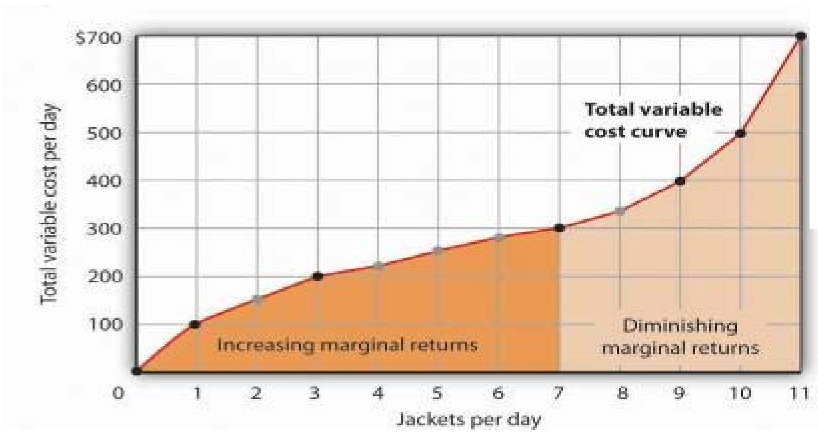
From BC let production function be $Q=L^{½}$

$dQ/dL = MP_L = ½L^{-1/2}$
$d^2Q/dL^2 = -1/4\ L^{-3/2} < 0$
Decreasing marginal returns

TVC = $wQ^2 = Q^2$ Let w = 1
$dTVQ/dQ = 2Q$

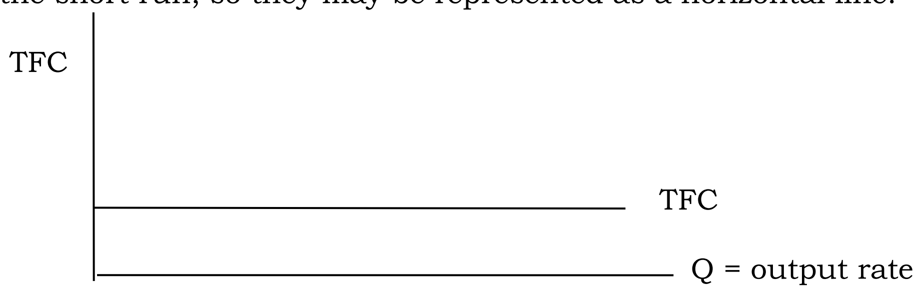
$d^2TVQ/dQ^2 = 2 > 0$
Marginal Cost is upward sloping. TVC rises at an increasing rate

It is easy to translate the quantity of the variable input into total variable cost (TVC): just multiply by the price of the input - the shape doesn't change. However, now we would like to express TVC as a function of the output of the product. This is easy: just flip the axes and we have the TVC curve:



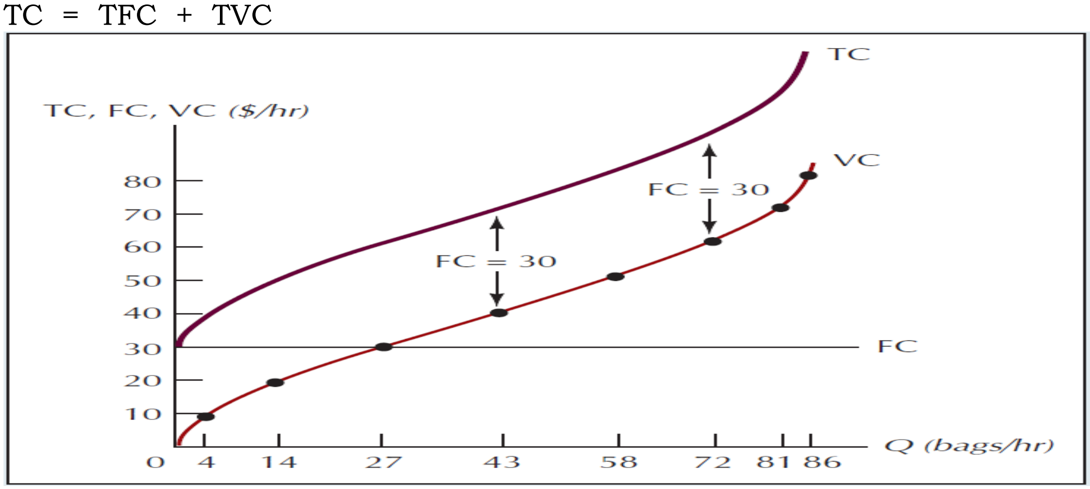
Total Variable Costs are the costs associated with hiring various levels of variable inputs in order to vary the rate of output in the short run."

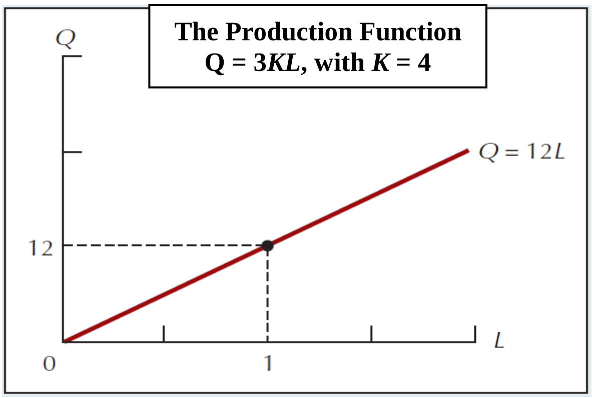
Now we need to talk about Total Fixed Costs (TFC). The Fixed Costs do not change as output varies in the short run, so they may be represented as a horizontal line:

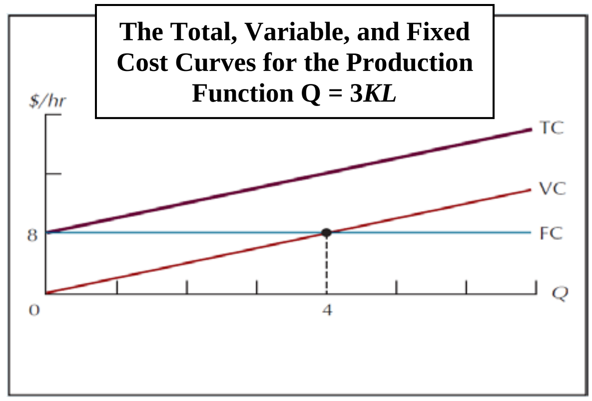


Finally, recall that the firm's Total Cost of producing output (TC) is the sum of its Total fixed cost (TFC) and it's Total Variable Cost (TVC):

TC = TFC + TVC







But, what the firm really needs to know is how the costs are distributed across the individual units of outputs and how they change when the level of output is increased or decreased. Thus, we want to look at the shapes of Average Total Cost (ATC), Average Variable Cost (AVC), Average Fixed Cost (AFC), and Marginal Cost (MC).

Average Fixed Cost (AFC) AFC = TFC / Q

Average Variable Cost (AVC)

AVC is the slope of the line from the origin to the point on the TVC function. This slope is a direct result of the law of diminishing marginal returns.

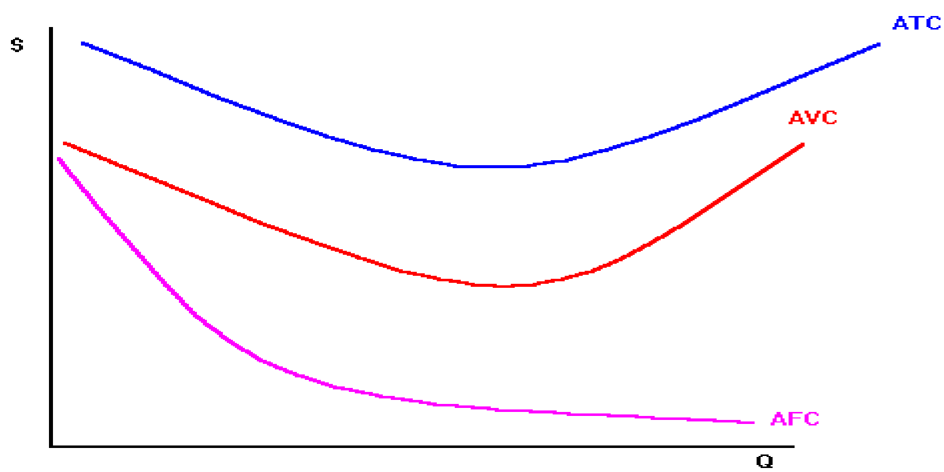
AVC = TVC /Q = wL /Q For simplicity assume w = 1

But AP_L = Q / L So AVC = 1 / AP_L.

As the AP_L fall, AVC rises and as AP_L rises, AVC falls. If AP_L is constant, AVC is constant.

Average Total Cost (ATC)

ATC = TC / Q = (TFC + TVC) / Q = AFC + AVC



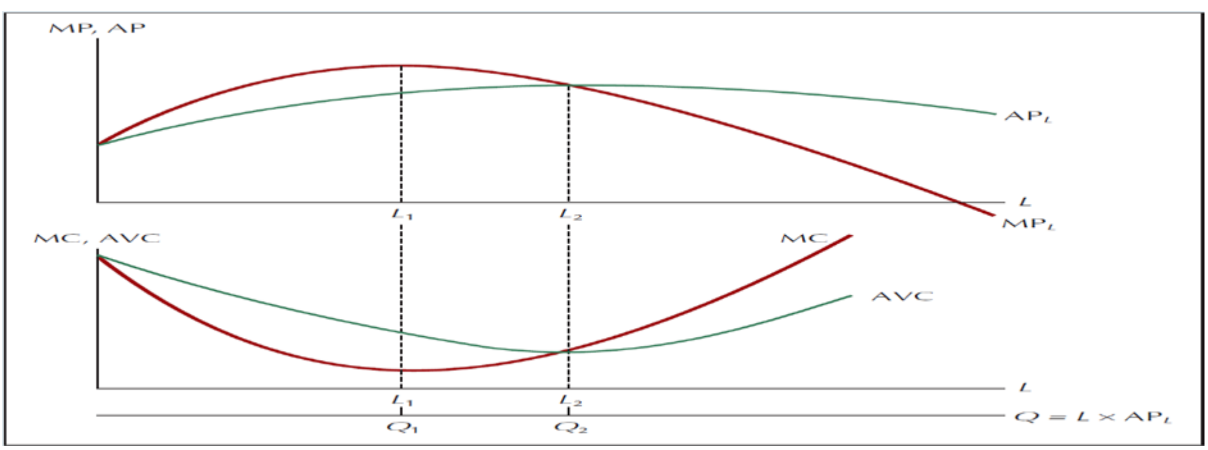
Marginal Cost (MC)

MC is the slope of TC. The shape is a direct result of the law of diminishing marginal returns.

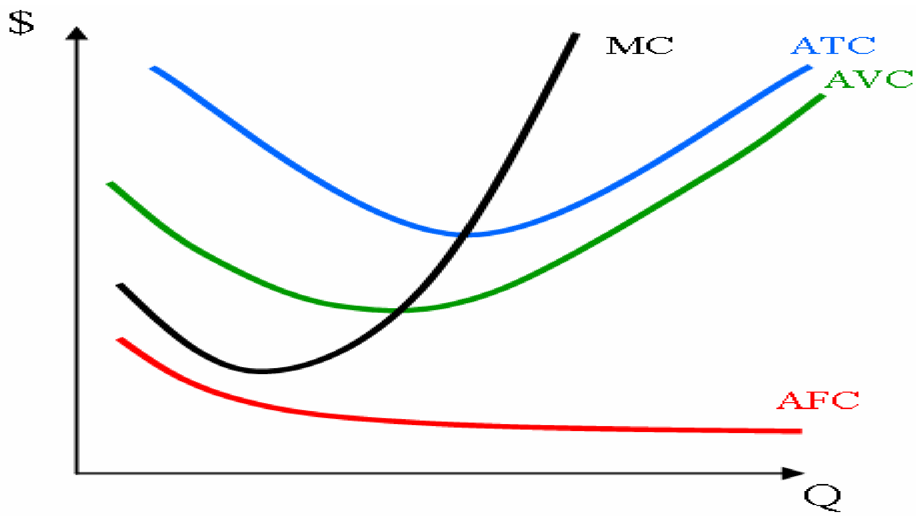
MC = ΔTVC / ΔQ = $w\,\Delta$L / ΔQ For simplicity assume w = 1

But $MP_L = \Delta Q / \Delta L$ So $MC = 1 / MP_L$.

The Relationship Between MP, AP, MC, and AVC



Consider the geometry of Average and Marginal Cost functions: Given the total cost function, we frequently want to derive the average and marginal cost functions. This can be done graphically in much the same way that we derived the **average** and **marginal** product curves. You can do this as an exercise. Your result should look like that below.



The Relationships Among Short-Run Cost Curves

1. ***AFC*** continuously declines and approaches both axes asymptotically.
2. ***AVC*** initially declines reaches a minimum and then increases.
3. When ***AVC*** is at a minimum it is equal to ***MC***.
4. ***ATC*** initially declines reaches a minimum and then increases.
5. When ***ATC*** is at a minimum it is equal to ***MC***.
6. ***MC*** is less than ***AVC*** and ***ATC*** when both curves are declining.
7. ***MC*** is greater than ***AVC*** and ***ATC*** when these curves are increasing.
8. ***MC*** equals ***AVC*** and ***ATC*** when both curves reach their minimum values.

MC passes through minimum of AC.

Proof:

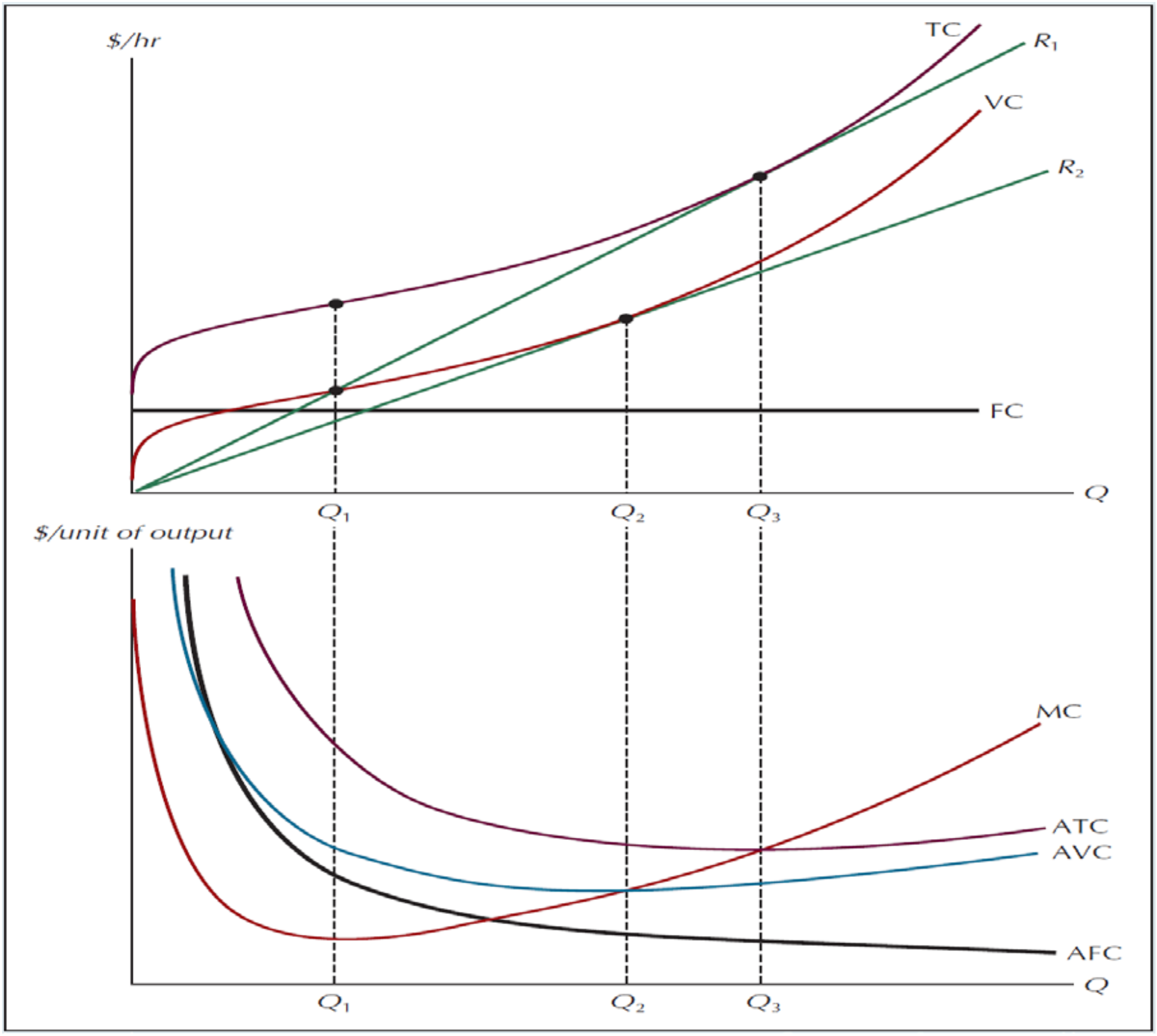
$$\frac{dAC}{dQ} = \frac{d\frac{TC}{Q}}{dQ} = \frac{(Q\frac{dTC}{dQ} - TC\frac{dQ}{dQ})}{Q^2} = \frac{Q(MC) - TC}{Q^2} = \frac{MC - AC}{Q} = 0$$

When AC is minimized MC = AC

Intuitive argument:
If MC >AC then extra units add more than proportionately to costs, so AC is rising
Similarly, if MC < AC, AC is falling

When MC = AC, AC is horizontal, i.e. it is at a minimum (or maximum

The Marginal, Average Total, Average Variable, and Average Fixed Cost Curves



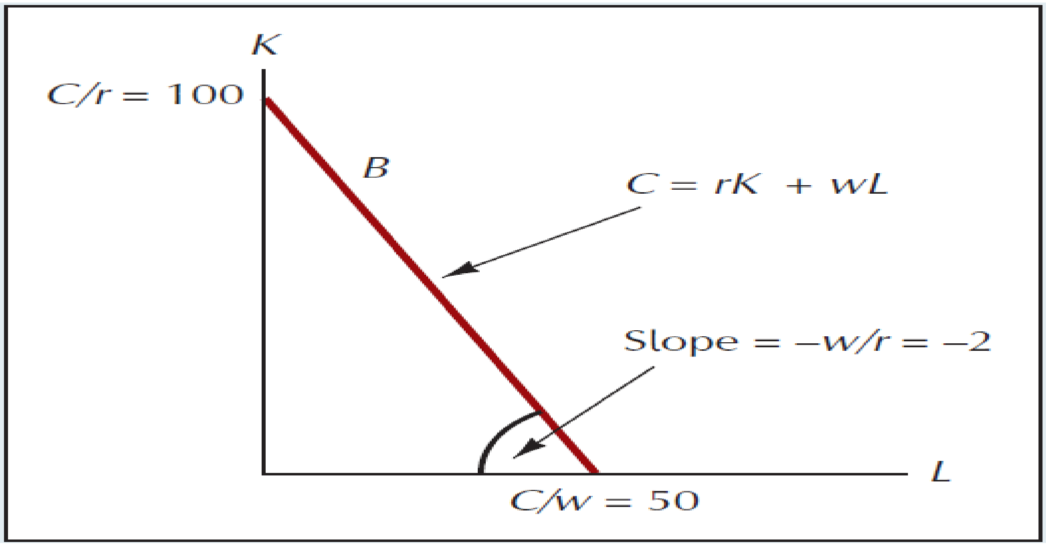
Cost Functions in the Long-Run

Isocosts - firms have an isocost that allows them to consider various combinations of inputs that yield the same total cost. As in the demand analysis unit earlier in the semester, we start with a budget equation:

$$C = wL + rK$$

Where w and r represent the Marginal factor cost (MFC) of L and K (assuming a competitive factor market – which means MFC is given) we can derive an equation of the line:

$K = \frac{C}{r} - \frac{w}{r}L$. The slope of the isocost is $-\frac{w}{r}$, which says that the price ratio of the inputs is equal to the slope of the isocost line.



The (Long Run) Cost Minimisation Problem

Suppose that a firm's owners wish to minimise costs of a given output (this is the same result of maximizing output for a given level of costs – the Duality concept)

Let the desired output be Q*

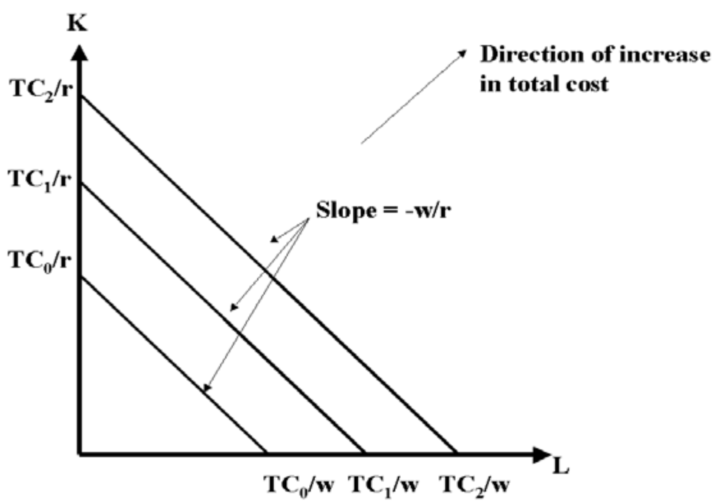
Q = Q(K,L)

Owner's problem: min TC = rK + wL Subject to Q* = Q(K, L)

A graphical solution

$TC^* = rK + wL$ or

$K = TC^*/r - (w/r)L$ is the the isocost line

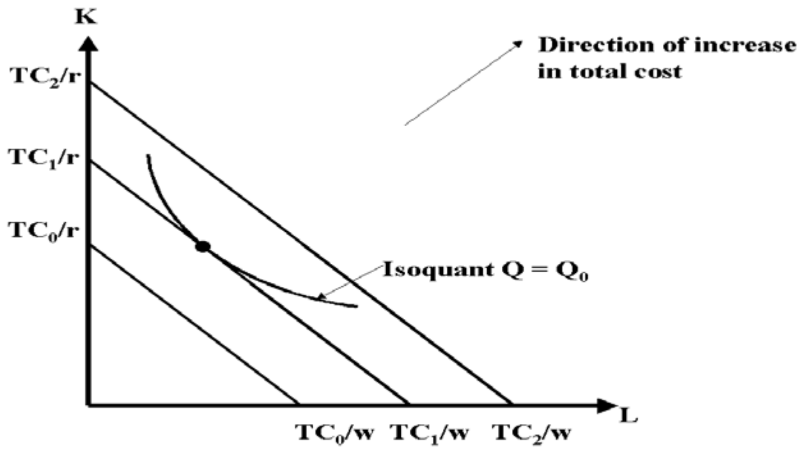


Cost minimisation subject to satisfaction of the isoquant equation: Q* = f(K, L)

Note: analogous to expenditure minimisation for the consumer

Tangency condition:

$MRTS_{L,K} = -MP_L/MP_K = -w/r$



The cost-minimizing input combination occurs where:

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

This can also be written as $\frac{MP_L}{w} = \frac{MP_K}{r}$

This is called the Marginal Rate of Technical Substitution (MRTS)

Mathematics of Cost-Minimization/Profit Maximization

Cost minimizing input choices. The above problem can be viewed as a profit maximization problem (where we select quantity). We show the analysis as a cost minimization problem, where K and L are selected, for a given output level, Q*.

This is a constrained minimization problem, hence, the Lagrangean function is:

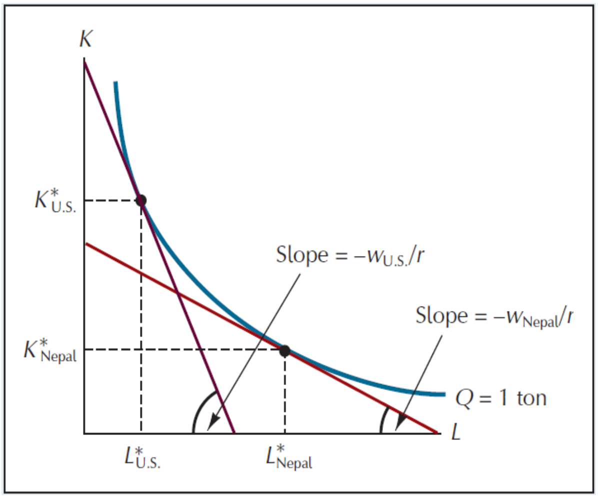
$$L' = wL + rK) + \lambda(Q^* - f(K,L)$$

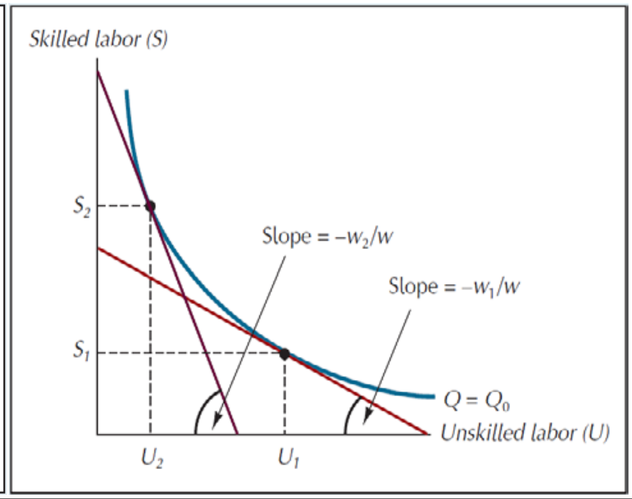
Taking the derivative of *L*' with respect to each good (i.e., *X* and *Y*) and setting it equal to zero:

$$\frac{\partial L'}{\partial L} = w - \lambda\frac{\partial Q}{\partial L} = 0 \qquad \frac{\partial L'}{\partial K} = r - \lambda\frac{\partial L'}{\partial K} = 0$$

Solving for λ: $\lambda = \frac{MP_L}{w}$ and $\lambda = \frac{MP_K}{r}$

Equating these two equations yields the same result as the before: $\frac{MP_L}{w} = \frac{MP_K}{r}$





Different Ways of Producing 1 Ton of Gravel

The Effect of a Minimum Wage Law on Unemployment of Skilled Labor

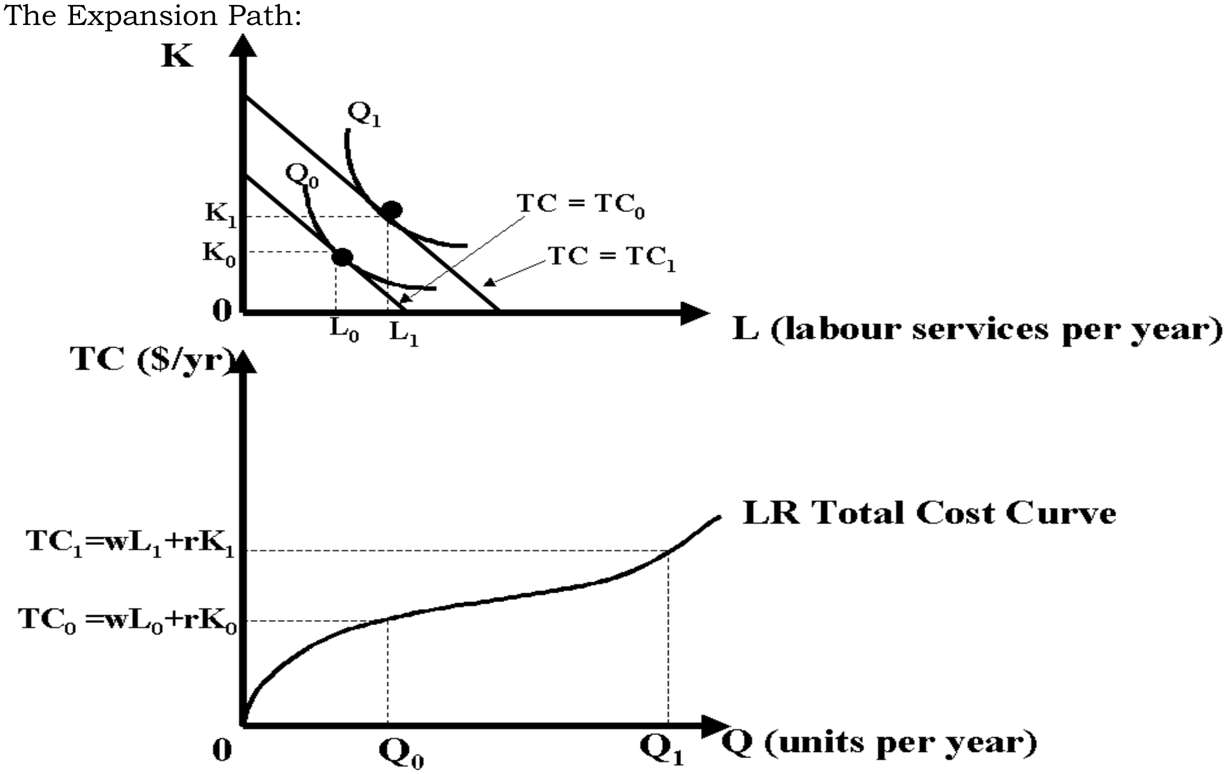
Example:

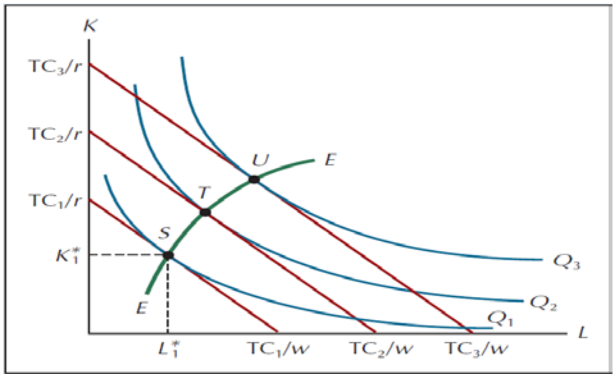
$Q = 50L^{1/2}K^{1/2}$ $MP_L = 25L^{-1/2}K^{1/2}$ $MP_K = 25L^{1/2}K^{-1/2}$

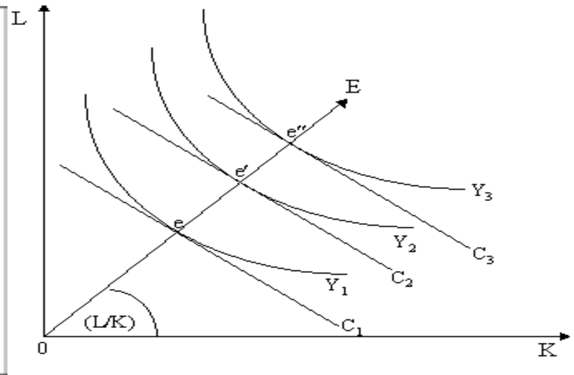
w = \$5 r = \$20 Q* = 1000 $MP_L/MP_K = K/L$ => K/L = 5/20…or…L=4K

$1000 = 50L^{1/2}K^{1/2}$ K = 10; L = 40

The Expansion Path:







Why is the expansion path a straight line?

Long Run Cost Curves

What we will look at now is the long run -- this where a firm can adjust ALL inputs in an optimal manner. In consumer theory -- indifference curves -- the analogous problem is how you adjust in response to the prices of goods changing. If the prices of X and Y you adjust the combination of goods that you buy moving to another optimal bundle. Well, a firm does the same thing, it's buying labor and capital and if the prices of those things change, then a firm readjusts how many people it hires, how many computers to use, etc.

Let's take a firm that has the following production function: Q = f(K, L). And it can buy all the labor it wants at w and all the capital it wants at r.

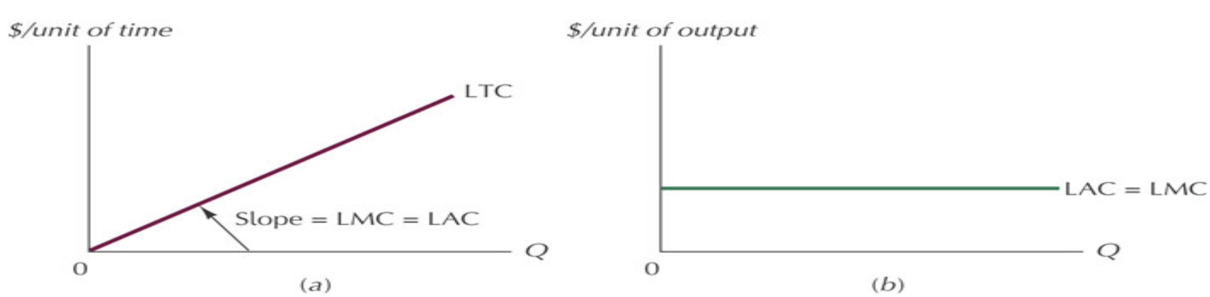
Therefore, total costs are equal to: $C = wL + rK$.

The first curve we'll talk about is the Average cost curve; obviously ATC = TC/Q.

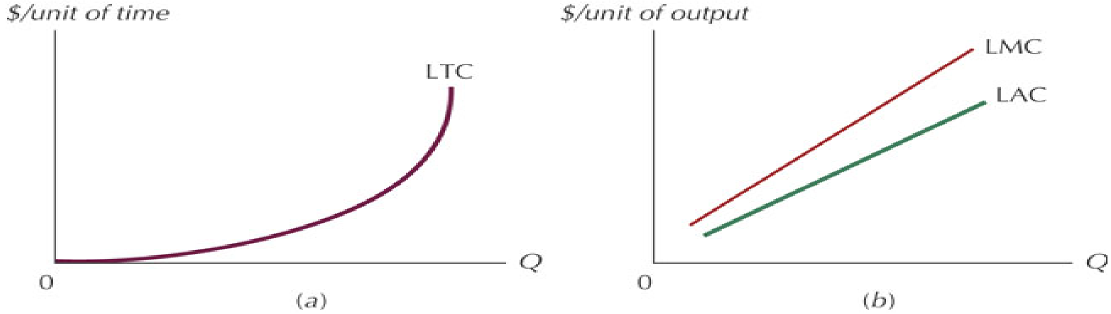
We now turn to the concept of **Economies and Diseconomies of Scale**. Remember we had the concept of diminishing marginal product of labor -- in that situation, the only thing that changed was labor, we hold the amount of capital constant. EOS deals with the situation of what happens to output, when we change everything.

We know we can have:

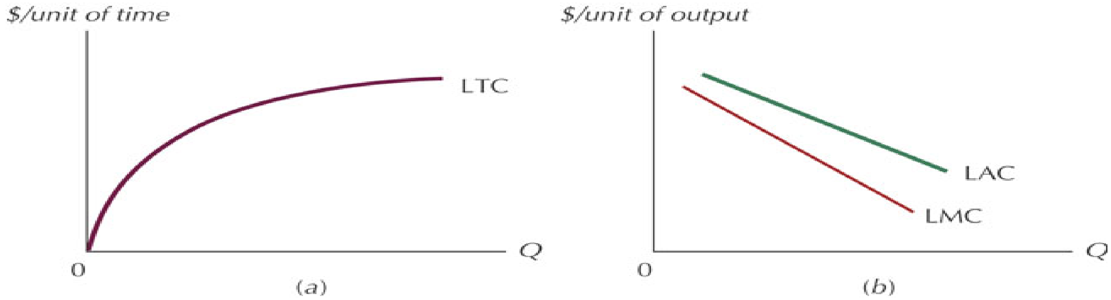
Constant Returns to Scale - long-run total costs are thus exactly proportional to output.



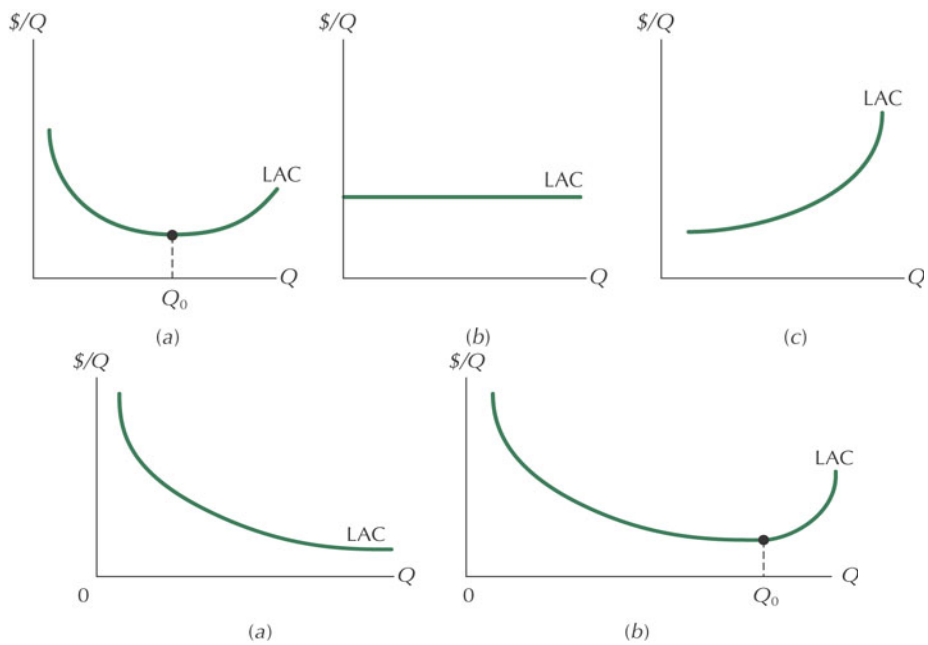
Decreasing returns to Scale or Diseconomies of Scale - a given proportional increase in output requires a greater proportional increase in all inputs and hence a greater proportional increase in costs.



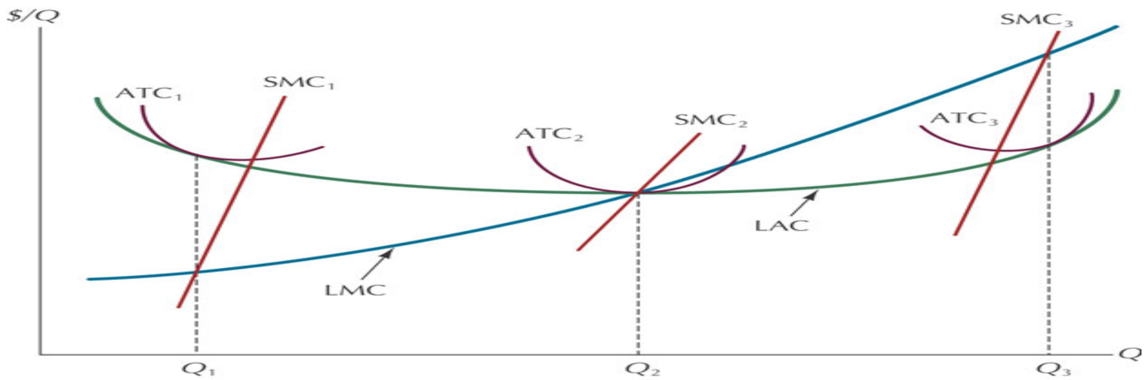
Increasing Rreturns to Scale or Economies of Scale - long-run total cost rises less than in proportion to increases in output.



Typically a firm experiences all three. In fact they usually have an ATC curve that looks like this:



The minimum point on the LRAC curve is we call the **optimal size or output of the firm**.



Example: Consider the following Total Cost curve equation:

Assume this is the total cost curve of a firm: TC = 145Q - 18Q^2 + 3Q^3. What is the optimal output of the firm?

To answer that question we need to know the average cost curve and the minimum point on that curve. This can be found two ways. We know two things about the ATC curve at the optimal point.

One, which is where the MC curve intersects the ATC.

Two, we know the slope of the ATC curve is zero at that point.

First, what is the equation for the ATC curve: $ATC = \frac{TC}{Q} = 145 - 18Q + 3Q^2$

The minimum point is where the ATC curve has a slope of zero or where the derivative with respect to Q is equal to zero (you can assume the appropriate second order conditions hold). Therefore,

$\frac{dATC}{dQ} = -18 + 6Q$ Set that to zero and solve for Q:

$$6Q - 18 = 0$$
$$6Q = 18$$
$$Q = 3$$

We can check our result by finding the MC curve, setting that equal to ATC curve, and solving for Q. Marginal cost is the derivation of the TC curve with respect to Q, that gives us the following equation:

$$MC = \frac{dTC}{dQ} = 145 - 36Q + 9Q^2 = 145 - 18Q + 3Q^2$$

$$18Q - 6Q^2 = 0$$

Solving for Q gives Q=3, confirming our original result. Therefore, the optimal output of the firm is 3 units. At 3 units, ATC = $118.