Chapter 5 Lecture - Discounted Cash Flow Valuation

Learning Objectives

After studying this chapter, you should be able to:

- **LO1** Determine the future and present value of investments with multiple cash flows.
- **LO2** Calculate loan payments and find the interest rate on a loan.
- **LO3** Describe how loans are amortized or paid off.
- **LO4** Explain how interest rates are quoted (and misquoted).

5-2

Future Value: Multiple Cash Flows (1)

- You think you will be able to deposit $4,000 at the end of each of the next three years in a bank account paying 8 percent interest.
- You currently have $7,000 in the account.
- How much will you have in 3 years?
- How much in 4 years?

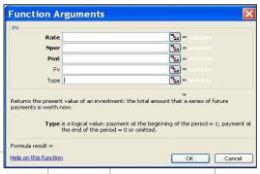
5-3

Future Value: Multiple Cash Flows (1)

- Find the value at year 3 of each cash flow and add them together.
  - Year 0: FV = $7,000(1.08)^3 = $ 8,817.98
  - Year 1: FV = $4,000(1.08)^2 = $ 4,665.60
  - Year 2: FV = $4,000(1.08)^1 = $ 4,320.00
  - Year 3: value = $ 4,000.00
  - Total value in 3 years = $21,803.58
- Value at year 4 = $21,803.58(1.08) = $23,547.87

5-4

FV Excel Solution



Excel Solution						
	Year	Nper	Rate	PV	PMT	FV
	0	3	0.08	-7000	0	8,817.98
	1	2	0.08	-4000	0	4,665.60
	2	1	0.08	-4000	0	4,320.00
	3					4,000.00
						21,803.58
	Value at year 4:					
	Year	Nper	Rate	PV	PMT	FV
	4	1	0.08	-21,803.58	0	23,547.87

=FV(Rate, Nper,PMT,PV)

5-5

FV Example Calculator Solution

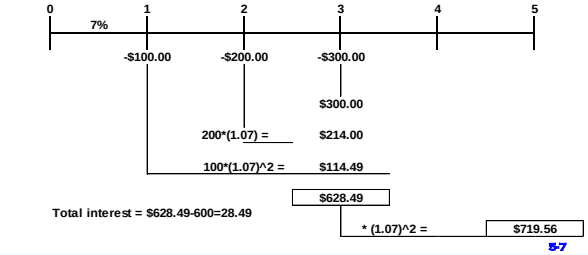
Calculator Solution						
	Year	N	I/Y	PV	PMT	CPT FV
	0	3	8	-7000	0	8,817.98
	1	2	8	-4000	0	4,665.60
	2	1	8	-4000	0	4,320.00
	3					4,000.00
						21,803.58
	Value at year 4:					
	Year	N	I/Y	PV	PMT	CPT FV
	4	1	8	-21,803.58	0	23,547.87

5-6

Future Value: Multiple Cash Flows (2)

- If you deposit $100 in one year, $200 in two years and $300 in three years.
- How much will you have in three years at 7 percent interest?

TIMELINE



5-7

Future Value: Multiple Cash Flows (2)

- If you deposit $100 in one year, $200 in two years and $300 in three years.
- How much will you have in three years at 7 percent interest?
- How much in five years if you don't add additional amounts?
  - Year 1 CF: 2 N; -100 PV; 7 I/Y; CPT FV = 114.49
  - Year 2 CF: 1 N; -200 PV; 7 I/Y; CPT FV = 214.00
  - Year 3 CF: 0 N; -300 PV; 7 I/Y; CPT FV = 300.00
  - Total FV_3 = 628.49
  - Total FV_5 = 628.49 * $(1.07)^2$ = 719.56

5-8

Future Value: Multiple Cash Flows (2)

Excel Solution

Rate	7%			
Year	Nper	CF	FV	Function
1	2	-100	$114.49	=FV(0.07,2,0,-100)
2	1	-200	$214.00	=FV(0.07,1,0,-200)
3	0	-300	$300.00	=FV(0.07,0,0,-300)
Total FV at Year 3			$628.49	
Total FV at Year 5			$719.56	=(628.49)*(1.07)^2

5-9

Future Value: Multiple Cash Flows (3)

- Suppose you invest $500 in a mutual fund today and $600 in one year.
- If the fund pays 9% annually, how much will you have in two years?

FV = $ 500 x $(1.09)^2$ = $ 594.05
+ $ 600 x (1.09) = $ 654.00
= $1,248.05

- How much will you have in 5 years if you make no further deposits?

$FV = \$500(1.09)^5 + \$600(1.09)^4 = \$1,616.26$

- Second way – use value at year 2:
- $FV = \$1,248.05(1.09)^3 = \$1,616.26$

Excel Solution

=FV(Rate, Nper,PMT,PV)

Year	Nper	Rate	PV	PMT	FV
0	2	0.09	-500	0	= 594.05
1	1	0.09	-600	0	= 654.00
					=1,248.05
Value at year 4:					
Year	Nper	Rate	PV	PMT	FV
5	3	0.09	-1,248.05	0	=1,616.26

5-10

FV Example Calculator Solution

Calculator Solution

					CPT
Year	N	I/Y	PV	PMT	FV
0	2	9	500	0	594.05
1	1	9	600	0	654.00
					1,248.05
Value at year 4:					CPT
Year	N	I/Y	PV	PMT	FV
5	3	9	1,248.05	0	1,616.26
or					CPT
Year	N	I/Y	PV	PMT	FV
0	5	9	500	0	769.31
1	4	9	600	0	846.95
					1,616.26

5-11

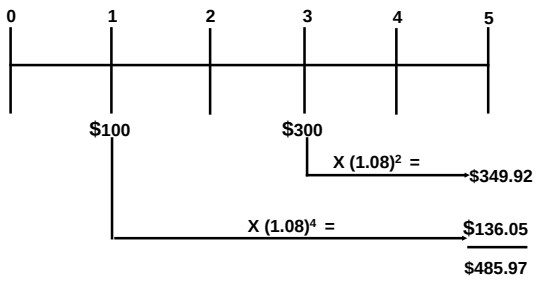
Future Value: Multiple Cash Flows Another Example - Formula

- Suppose you plan to deposit $100 into an account in one year and $300 into the account in three years.
- How much will be in the account in five years if the interest rate is 8%?

$FV = \$100(1.08)^4 + \$300(1.08)^2 = \$136.05 + \$349.92 = \$485.97$

5-12

Example Time Line



5-13

FV Calculator and Excel Solution

Calculator Solution					CPT
Year	N	I/Y	PV	PMT	FV
1	4	8	-100	0	136.05
3	2	8	-300	0	349.92
					485.97
Excel Solution					
Year	Nper	Rate	PV	PMT	FV
1	4	0.08	-100	0	136.05
3	2	0.08	-300	0	349.92
					485.97

=FV(Rate, Nper,PMT,PV)

5-14

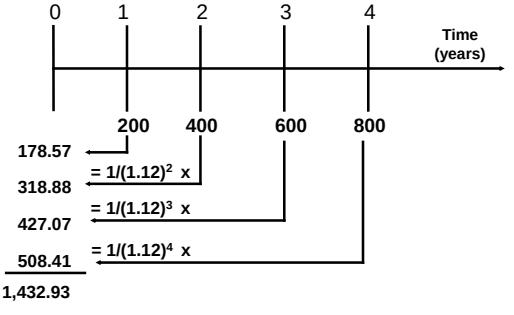
Present Value: Multiple Cash Flows

Find the PV of each cash flow and add them:

- **Year 1 CF: $200 / (1.12)¹ = $ 178.57**
- **Year 2 CF: $400 / (1.12)² = $ 318.88**
- **Year 3 CF: $600 / (1.12)³ = $ 427.07**
- **Year 4 CF: $800 / (1.12)⁴ = $ 508.41**
- **Total PV = $1,432.93**

5-15

Time Line



5-16

Multiple Uneven Cash Flows Using the TI BAII's Cash Flow Worksheet

- Clear all:
  - Press *CF*
  - Then *2nd*
  - Then *CE/C*
- CF_0 is displayed as 0.00
- Enter the Period 0 cash flow
  - If an outflow, press +/– to change the sign
- To enter the figure in the cash flow register, press *ENTER*

5-17

TI BAII+: Uneven Cash Flows

- Press the down arrow to move to the next cash flow register
- Enter the cash flow amount, press *ENTER* and the down arrow to move to the cash flow counter (F0n)
- The default counter value is "1"
  - To accept the value of "1", press the down arrow again
  - To change the counter, enter the correct count, press *ENTER* and then the down arrow

5-18

TI BAII+: Uneven Cash Flows

- Repeat for all cash flows, in order.
- To find NPV:
  - Press *NPV*: I appears on the screen.
  - Enter the interest rate, press *ENTER*, and then the down arrow to display NPV.
  - Press *CPT*.

5-19

TI BAII+: Uneven Cash Flows

Cash Flows:		
CF0	=	0
CF1	=	200
CF2	=	400
CF3	=	600
CF4	=	800

Display	You Enter		
	CF	2^{nd}	CE/C
C00	0 ENTER		
C01	200 ENTER		
F01	1 ENTER		
C02	400 ENTER		
F02	1 ENTER		
C03	600 ENTER		
F03	1 ENTER		
C04	800 ENTER		
F04	1 ENTER		NPV
I	12 ENTER		down
NPV	CPT		
1432.93			

5-20

PV of Multiple Uneven CFs – Calculator and Excel Solutions

Excel Solution

Rate		12%		
Period	Cash Flow		Present Value	Formula
1	$	200.00	($178.57)	=PV(B1,A3,0,B3)
2	$	400.00	($318.88)	=PV(B1,A4,0,B4)
3	$	600.00	($427.07)	=PV(B1,A5,0,B5)
4	$	800.00	($508.41)	=PV(B1,A6,0,B6)
	Total PV =		($1,432.93)	=SUM(C3:C6)
			($1,432.93)	=-NPV(B1,B3:B6)

The functions require a PMT = 0.

Calculator Solution

Year	N	I/Y	FV	PMT	CPT PV
1	1	12	200	0	178.57
2	2	12	400	0	318.88
3	3	12	600	0	427.07
4	4	12	800	0	508.41
					1,432.93

5-21

Present Value: Multiple Cash Flows Another Example – Formula Solution

- You are considering an investment that will pay you $1,000 in one year, $2,000 in two years and $3,000 in three years.
- If you want to earn 10% on your money, how much would you be willing to pay?
  - PV = $1,000 / $(1.1)^1$ = $ 909.09
  - PV = $2,000 / $(1.1)^2$ = $1,652.89
  - PV = $3,000 / $(1.1)^3$ = $2,253.94
  - PV = $4,815.92

5-22

Multiple Cash Flows – PV Example with Excel and Calculator Solutions

Calculator Solution						CPT
	Year	N	I/Y	FV	PMT	PV
	1	1	10	-1000	0	909.09
	2	2	10	-2000	0	1,652.89
	3	3	10	-3000	0	2,253.94
						4,815.92
Excel Solution						
	Year	Nper	Rate	FV	PMT	PV
	1	1	0.10	-1000	0	909.09
	2	2	0.10	-2000	0	1,652.89
	3	3	0.10	-3000	0	2,253.94
						4,815.92
	=PV(Rate, Nper,PMT,FV)					

5-23

Present Value: Multiple Cash Flows Another Example – Formula Solution

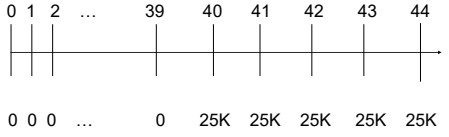
- You are considering an investment that will pay you $1,000 in one year, $2,000 in two years and $3,000 in three years.
- If you want to earn 10% on your money, how much would you be willing to pay?
  - PV = $1,000 / $(1.1)^1$ = $ 909.09
  - PV = $2,000 / $(1.1)^2$ = $1,652.89
  - PV = $3,000 / $(1.1)^3$ = $2,253.94
  - PV = $4,815.92

5-24

Saving For Retirement

You are offered the opportunity to put some money away for retirement. You will receive five annual payments of $25,000 each beginning in 40 years. How much would you be willing to invest today if you desire an interest rate of 12%?

Timeline



Notice that the year 0 cash flow = 0 (CF_0 = 0)

Cash flows years 1–39 = 0 Cash flows years 40–44 = 25,000

Show the answer is $1,084.71

5-25

Excel and Calculator Solution

Nper	0-39	0	Rate	0.12	
Nper	40	-25000			$268.67
Nper	41	-25000			$239.88
Nper	42	-25000			$214.18
Nper	43	-25000			$191.23
Nper	44	-25000			$170.74
				PV=	$1,084.71

Use cash flow keys:

	CF
	2nd CE/C
CF0	*0 ENTER*
C01	*0 ENTER*
F01	*39 ENTER*
C02	*25000 ENTER*
F02	*5 ENTER*
	NPV
I	*12 ENTER*
	DOWN
	CPT

1084.71

5-26

Decisions, Decisions

- **Your broker calls you and tells you that he has this great investment opportunity.**
- **If you invest $100 today, you will receive $40 in one year and $75 in two years.**
- **If you require a 15% return on investments of this risk, should you take the investment?**

Excel Solution

Use Formula PV(Rate,Nper,Pmt,FV)

PV = PV(0.15, 1, 0, 40) = $34.78

+ PV = PV(0.15, 2, 0, 75) = $56.71

= $91.49

Don't take the investment – broker is charging more than you would be willing to pay.

Use cash flow keys:

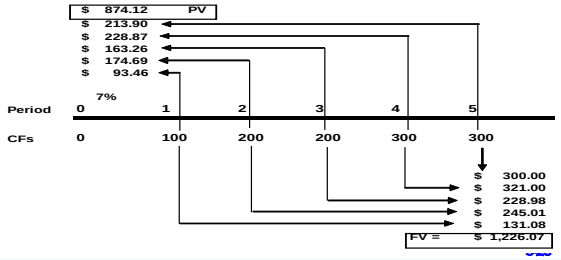
	CF
	2nd CE/C
CF0	*0 ENTER*
C01	*40 ENTER*
F01	*1 ENTER*
C02	*75 ENTER*
F02	*1 ENTER*
	NPV
I	*15 ENTER*
	DOWN CPT

91.49

5-27

Quick Quiz

- **Suppose you are looking at the following possible cash flows:**
  - **Year 1 CF = $100;**
  - **Years 2 and 3 CFs = $200;**
  - **Years 4 and 5 CFs = $300.**
  - **The required discount rate is 7%**
- **What is the value of the CFs at year 5?**
- **What is the value of the CFs today?**



Quick Quiz – Excel Solution

	Rate	7%		
Year	Nper	CF	PV	Formula
1	1	100	$93.46	=-PV(C2,A4,0,C4)
2	2	200	$174.69	=-PV(C2,A5,0,C5)
3	3	200	$163.26	=-PV(C2,A6,0,C6)
4	4	300	$228.87	=-PV(C2,A7,0,C7)
5	5	300	$213.90	=-PV(C2,A8,0,C8)
		Total PV	$874.17	=SUM(C4:C8)
Year	Nper	CF	FV	Year
1	4	100	$131.08	=-FV(C2,B12,0,C12)
2	3	200	$245.01	=-FV(C2,B13,0,C13)
3	2	200	$228.98	=-FV(C2,B14,0,C14)
4	1	300	$321.00	=-FV(C2,B15,0,C15)
5	0	300	$300.00	=-FV(C2,B16,0,C16)
		Total FV	$1,226.07	=SUM(C12:C16)

Annuities and Perpetuities

- Annuity – finite series of equal payments that occur at regular intervals
  - If the first payment occurs at the end of the period, it is called an *ordinary annuity*
  - If the first payment occurs at the beginning of the period, it is called an *annuity due*
- Perpetuity – infinite series of equal payments.

Annuities and Perpetuities Basic Formulas

- Perpetuity: PV = PMT / r
- Annuities:

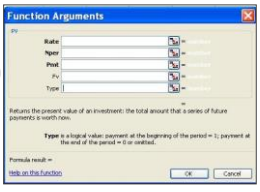
$$PV = PMT\left[\frac{1 - \frac{1}{(1+r)^t}}{r}\right]$$

$$FV = PMT\left[\frac{(1+r)^t - 1}{r}\right]$$

Derivation of Annuity Formula
http://www.ftsmodules.com/public/texts/bondtutor/appendix1a.htm

Excel Spreadsheet Functions

- FV(Rate,Nper,Pmt,PV,*0/1*)
- PV(Rate,Nper,Pmt,FV,*0/1*)
- RATE(Nper,Pmt,PV,FV,*0/1*)
- NPER(Rate,Pmt,PV,FV,*0/1*)
- PMT(Rate,Nper,PV,FV,*0/1*)



- Inside parenthess: (RATE,NPER,PMT,PV,FV,*0/1*)
- *"0/1" Ordinary annuity = 0 (default; no entry needed)*
- *Annuity Due = 1 (must be entered)*

Important Points to Remember

- Interest rate and time period must match!
  - Annual periods ⇒ annual rate
  - Monthly periods ⇒ monthly rate
- The Sign Convention
  - Cash inflows are positive
  - Cash outflows are negative

5-33

Sign Convention Example Calculator

5	N
10	I/Y
-100	PV
20	PMT
CPT FV = $38.95	

Implies you deposited $100 today and plan to WITHDRAW $20 a year for 5 years

5	N
10	I/Y
-100	PV
-20	PMT
CPT FV = $283.15	

Implies you deposited $100 today and plan to ADD $20 a year for 5 years

+CF = Cash INFLOW to YOU

-CF = Cash OUTFLOW from you

5-34

Annuity

- You can afford $632 per month.
- Going rate = 1%/month for 48 months.
- How much can you borrow?
- You borrow money TODAY so you need to compute the present value.

48 *N*
1 *I/Y*
632 *PMT*
0 *FV*
CPT PV = 23,999.54
($24,000)

$$PV = 632\left[\frac{1-\frac{1}{(1.01)^{48}}}{.01}\right] = 23{,}999.54$$

Excel Solution

=PV(0.01,48,-632,0)

Rate	0.01		
Nper	48		
Pmt	-632		
		PV=	$23,999.54

5-35

Annuity – Sweepstakes Example

- Suppose you win the Publishers Clearinghouse $10 million sweepstakes.
- The money is paid in equal annual installments of $333,333.33 over 30 years.
- If the appropriate discount rate is 5%, how much is the sweepstakes actually worth today?
  - PV = $333,333.33[1 – 1/1.05^{30}] / .05 = $5,124,150.29

Excel Solution

=PV(5, 30, 333333.33, 0)

Rate	0.05		
Nper	30		
Pmt	-333,333.33		
Type	0		
		PV =	$5,124,150.29

N	30
I/Y	5
PMT	$ 333,333.33
FV	0
CPT	PV
$ (5,124,150.29)	

5-36

Buying a House

- You are ready to buy a house and you have $20,000 for a down payment and closing costs.
- Closing costs are estimated to be 4% of the loan value.
- You have an annual salary of $36,000.
- The bank is willing to allow your monthly mortgage payment to be equal to 28% of your monthly income.
- The interest rate on the loan is 6% per year with monthly compounding (.5% per month) for a 30-year fixed rate loan.
- How much money will the bank loan you?
- How much can you offer for the house?

5-37

Buying a House – Calculator Solution

- Bank loan
  - Monthly income = 36,000 / 12 = 3,000
  - Maximum payment = .28(3,000) = 840
    - 360 N (30*12)
    - 0.5 I/Y
    - -840 PMT
    - CPT PV = 140,105
- Total Price
  - Closing costs = .04(140,105) = 5,604
  - Down payment = 20,000 – 5604 = 14,396
  - Total Price = 140,105 + 14,396 = 154,501

5-38

Buying a House - Continued

- Bank loan
  - Monthly income = 36,000 / 12 = 3,000
  - Maximum payment = .28(3,000) = 840

Excel Solution

=PV(.005,360,-840,0)

Rate	.06/12	0.005		
Nper	12*30	360		
Pmt	-840	-840.00		
			Loan (PV)=	$140,104.96

- Total Price
  - Closing costs = .04(140,105) = 5,604
  - Down payment = 20,000 – 5604 = 14,396
  - Total Price = 140,105 + 14,396 = 154,501

5-39

Quick Quiz

- You know the payment amount for a loan and you want to know how much was borrowed.
  - Do you compute a present value or a future value?
- Example - You want to receive $5,000 per month in retirement. If you can earn .75% per month and you expect to need the income for 25 years, how much do you need to have in your account at retirement?

= -595,808.11

Excel Solution

=PV(0.0075,300,5000,0)

Rate	0.0075			
Nper	300			
Pmt	5000			
			Amount Needed (PV)	($595,808.11)

5-40

Finding the Payment

- Suppose you want to borrow $20,000 for a new car.
- You can borrow at 8% per year, compounded monthly (8/12 = .66667% per month).
- If you take a 4 year loan, what is your monthly payment?

Excel Solution

=PMT(0.006667,48,20000,0)

Rate	0.00666667	
Nper	48	
Pv	20000	
	Monthly Payment (PMT)=	($488.26)

4(12) = 48 N
0.66667 I/Y
20,000 PV
0 FV
CPT PMT = -488.26

5-41

Finding the Number of Payments Example

- $1,000 due on credit card
- Payment = $20 month minimum
- Rate = 1.5% per month
- The sign convention matters!!!

Excel Solution

=NPER(0.015,-20,1000,0)

Rate	0.015		
Pmt	-20		
Pv	1000		
	NPER =	93.11	(months)
	Years =	7.759254	

1.5 I/Y
1000 PV
-20 PMT
0 FV
CPT N = 93.111 months
= 7.75 years

5-42

Finding the Number of Payments Another Example

Suppose you borrow $2,000 at 5% and you are going to make annual payments of $734.42. How long before you pay off the loan?

Excel Solution

=NPER(0.05,-734.42,2000,0)

Rate	0.05		
Pmt	-734.42		
Pv	2000		
	NPER=	2.999987	= 3 years

5-43

Finding the Rate

Suppose you borrow $10,000 from your parents to buy a car. You agree to pay $207.58 per month for 60 months. What is the monthly interest rate?

Excel Solution

=RATE(60,-207.58,10000,0)

Nper	60	
Pmt	-207.58	
Pv	10000	
	RATE =	0.00749939
		0.75% Per month

5-44

Quick Quiz

You want to receive $5,000 per month for the next 5 years. How much would you need to deposit today if you can earn .75% per month?

Excel Solution

=PV(0.0075,60,5000,0)

300 N Months
0.75 I/Y ⬅ Monthly rate
5000 PMT ⬅ Monthly Payment
0 FV
CPT PV = -595,808.11

Rate	0.0075	
Nper	60	
Pmt	5000	
	PV=	($240,866.87)

5-45

Quick Quiz

- **You want to receive $5,000 per month for the next 5 years.**
- **What monthly rate would you need to earn if you only have $200,000 to deposit?**

Excel Solution

=RATE(60,5000,-200000,0)

60 N
-200000 PV
5000 PMT
0 FV
CPT I/Y = 1.4395% per month

Nper	60		
Pmt	5000		
Pv	-200000		
		RATE=	0.014394781
		=	1.439% per month

5-46

Quick Quiz

- **Suppose you have $200,000 to deposit and can earn .75% per month.**
  - **How many months could you receive the $5,000 payment?**

Excel Solution

=NPER(0.0075,5000,-200000,0)

0.75 I/Y
-200000 PV
5000 PMT
0 FV
CPT N = 47.73 months
≈ 4 years

Rate	0.0075		
Pmt	-5000		
Pv	200000		
		NPER=	47.73477457
		=	48 months or 4 years

5-47

Quick Quiz

- **Suppose you have $200,000 to deposit and can earn .75% per month.**
- **How much could you receive every month for 5 years?**

Excel Solution

=PMT(0.0075,60,-200000,0)

60 N
0.75 I/Y
-200000 PV
0 FV
CPT PMT = 4151.67

Rate	0.0075		
Nper	60		
Pv	-200000		
		PMT=	$4,151.67

5-48

Future Values for Annuities

Suppose you begin saving for your retirement by depositing $2,000 per year in an IRA. If the interest rate is 7.5%, how much will you have in 40 years?

$$FV = PMT\left[\frac{(1+r)^t - 1}{r}\right]$$

$$FV = 2000\left[\frac{(1.075)^{40} - 1}{.075}\right] = 454{,}513.04$$

5-49

Excel and Calculator Solution

Excel Solution

=FV(0.075,40,-2000,0)

Rate	0.075		
Nper	40		
Pmt	2000		
		FV=	($454,671.84)

40 N
7.5 I/Y
0 PV
-2000 PMT
CPT FV = 454513.04

5-50

Annuity Due

You are saving for a new house and you put $10,000 per year in an account paying 8%. The first payment is made today. How much will you have at the end of 3 years?

$$FV_{AD} = PMT\left[\frac{(1+r)^t - 1}{r}\right](1+r)$$

$$FV_{AD} = 10000\left[\frac{(1.08)^3 - 1}{.08}\right](1.08) = 35{,}061.12$$

5-51

Excel and Calculator Solution

Excel Solution

=FV(0.08,3,-10000,0,1)

Rate	0.08		
Nper	3		
Pmt	-10000		
Type	1	FV	$35,061.12

2nd BGN 2nd SET
3 N
8 I/Y
0 PV
-10000 PMT
CPT FV = 35061.12
2nd BGN 2nd SET

5-52

Quick Quiz

You want to have $1 million to use for retirement in 35 years. If you can earn 1% per month, how much do you need to deposit on a monthly basis if the first payment is made in one month?

Ordinary Annuity

Excel Solution

=PMT(0.01,420,0,1000000)

Rate	0.01		
Nper	420		
FV	1000000		
Type	0	PMT=	($155.50)

420	N
1	I/Y
0	PV
1000000	FV
CPT PMT = -155.50	

5-53

Quick Quiz

You want to have $1 million to use for retirement in 35 years. If you can earn 1% per month, how much do you need to deposit on a monthly basis if the first payment is made today?

Annuity Due

Excel Solution

=PMT(0.01,420,0,1000000,1)

Rate	0.01		
Nper	420		
FV	1000000		
Type	1	PMT=	($153.96)

2nd BGN 2nd SET	
420	N
1	I/Y
0	PV
1000000	FV
CPT PMT = -153.96	
2nd BGN 2nd SET	

5-54

Summary of Calculations

I. **Symbols**
PV = Present value, what future cash flows are worth today
FV_t = Future value, what cash flows are worth in the future at time t
r = Interest rate, rate of return, or discount rate per period—typically, but not always, one year
t = Number of periods—typically, but not always, the number of years
C = Cash amount

II. **Future value of C invested per period for t periods at r percent per period**
$FV_t = C \times [(1 + r)^t - 1]/r$
A series of identical cash flows paid for a set number of periods is called an annuity, and the term $[(1 + r)^t - 1]/r$ is called the *annuity future value factor.*

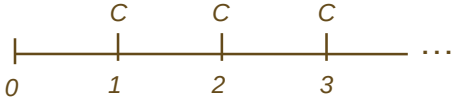
III. **Present value of C per period for t periods at r percent per period**
$PV = C \times \{1 - [1/(1 + r)^t]\}/r$
The term $\{1 - [1/(1 + r)^t]\}/r$ is called the *annuity present value factor.*

IV. **Present value of a perpetuity of C per period**
$PV = C/r$
A perpetuity has the same cash flow every period forever.

5-55

Perpetuity

A constant stream of cash flows that lasts forever



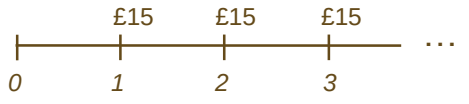
$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{r}$$

5-56

Perpetuity: Example

What is the value of a British consol that promises to pay £15 every year for ever?
The interest rate is 10-percent.

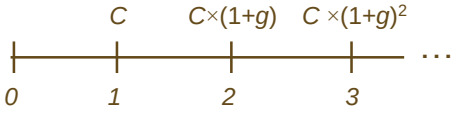


$$PV = \frac{£15}{.10} = £150$$

5-57

Growing Perpetuity

A growing stream of cash flows that lasts forever



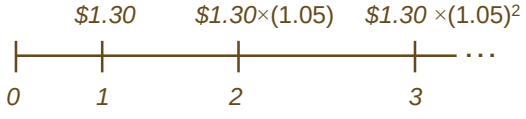
$$PV = \frac{C}{(1+r)} + \frac{C \times (1+g)}{(1+r)^2} + \frac{C \times (1+g)^2}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{r-g}$$

5-58

Growing Perpetuity: Example

- **The expected dividend next year is $1.30, and dividends are expected to grow at 5% forever.**
- **If the discount rate is 10%, what is the value of this promised dividend stream?**



$$PV = \frac{\$1.30}{.10 - .05} = \$26.00$$

5-59

Quick Quiz

You are considering a preferred stock that pays a quarterly dividend of $1.50. If your desired return is 3% per quarter, how much would you be willing to pay?

$$PV = \frac{C}{r}$$

$$PV = \frac{\$1.50}{.03} = \$50$$

5-60

Interest Rates

- Effective Annual Rate (EAR)
  - The interest rate expressed as if it were compounded once per year.
  - Used to compare two alternative investments with different compounding periods
- Annual Percentage Rate (APR) "Nominal"
  - The annual rate quoted by law
  - APR = periodic rate X number of periods per year
  - Periodic rate = APR / periods per year

5-61

Things to Remember

- You ALWAYS need to make sure that the interest rate and the time period match.
  - Annual periods ➔ annual rate.
  - Monthly periods ➔ monthly rate.
- If you have an APR based on monthly compounding, you have to use monthly periods for lump sums or adjust the interest rate accordingly.

5-62

EAR Formula

$$EAR = \left[1 + \frac{APR}{m}\right]^{m} - 1$$

APR = the quoted rate

m = number of compounds per year

5-63

EAR and APR in TI BA II+

2nd ICONV

2nd CE/C (to clear the memory)

- 3 fields in worksheet:
  - NOM (Nominal rate-APR)
  - EFF (Effective annual rate)
  - C/Y (Compounding periods/yr)
  - To compute EFF, enter the NOM and C/Y values, move to EFF and press *CPT*
  - To compute NOM, enter the EFF and C/Y values, move to NOM and press CPT

5-64

EAR and NOM in Excel

- 2 Functions:
  - =EFFECT(Nom, Nper)
  - =NOMINAL(Eff, Nper)
  - Note Nominal is the same as APR
- All rates entered as decimals
- Nper = number of compounding periods per year

Reminder TOOLS … Add-Ins … ANALYSIS TOOLPAK

5-65

Decisions, Decisions

- Which savings accounts should you choose:
  - 5.25% with daily compounding.
  - 5.30% with semiannual compounding.
- First account:
  - EAR = $(1 + .0525/365)^{365} - 1$ = 5.39%
  - Excel: =EFFECT(0.525,365) = 5.39%
- Second account:
  - EAR = $(1 + .053/2)^2 - 1$ = 5.37%
  - Excel: =EFFECT(0.53,2) = 5.37%

5-66

Computing APRs

- What is the APR if the monthly rate is .5%?
  - .5%(12) = 6%
- What is the APR if the semiannual rate is .5%?
  - .5%(2) = 1%
- What is the monthly rate if the APR is 12% with monthly compounding?
  - 12% / 12 = 1%
  - Can you divide the above APR by 2 to get the semiannual rate?
    - NO. You need an APR based on semiannual compounding to find the semiannual rate.

5-67

Computing EAR and APR

- Suppose you can earn 1% per month on $1 invested today.
  - What is the APR? 1(12) = 12%
  - How much are you effectively earning?
    - FV = $1(1.01)^{12}$ = 1.1268
    - Rate = (1.1268 – 1) / 1 = .1268 = 12.68%

Excel Solution

INCONV: NOM = 12
C/Y = 12
EFF = 12.6825

= EFFECT(0.12,12)

Nominal rate		
Npery		
	EFFECT=	0.12682503

5-68

Computing EAR and APR

- **Suppose if you put it in another account, you earn 3% per quarter.**
  - **What is the APR? 3(4) = 12%**
  - **How much are you effectively earning?**
    - **FV = $1(1.03)^4$ = 1.1255**
    - **Rate = (1.1255 – 1) / 1 = .1255 = 12.55%**

ICONV:	NOM	= 12
	C/Y	= 4
	EFF	= 12.5509

Excel Solution

=EFFECT(0.12,4)

Nominal rate	12		
Npery	12		
		EFFECT=	0.13

5-69

Frequency of Compounding

General Formula:

$$FV_t = PV_0\left(1 + \frac{r}{m}\right)^{mt}$$

t: Number of Years
m: Compounding Periods per Year
r: Annual Interest Rate or APR
FV_t: FV at the end of Year t
PV_0: PV of the Cash Flow today

5-70

Impact of Frequency

Ms. H has $1,000 to invest for 2 years at an annual interest rate of 12%.

Annual $FV_2 = 1{,}000(1 + [.12/1])^{(1)(2)} = 1{,}254.40$

Semi $FV_2 = 1{,}000(1 + [.12/2])^{(2)(2)} = 1{,}262.48$

Quarterly $FV_2 = 1{,}000(1 + [.12/4])^{(4)(2)} = 1{,}266.77$

Monthly $FV_2 = 1{,}000(1 + [.12/12])^{(12)(2)} = 1{,}269.73$

Daily $FV_2 = 1{,}000(1 + [.12/365])^{(365)(2)} = 1{,}271.20$

5-71

Continuous Compounding

Continuous compounding implies $m \to \infty$

If -- r = interest rate per period (one year)
m = number of periods per year
t = number of years

$$FV_t = PV_0\,(1 + r/m)^{mt} = PV_0\,\{1 + 1/(m/r)\}^{mt}$$

$$= PV_0\,[\{1 + 1/(m/r)\}^{m/r}]^{rt}$$

or

$$FV_t = PV_0\,[\{1 + 1/(m/r)\}^{m/r}]^{rt}$$

5-72

Solving the Equation

By definition $(1 + 1/n)^n$ as $n \to \infty$ is equal to e = 2.7182 . . .

If interest is compounded continuously or $m \to \infty$,

$\{1 + 1(m/r)\}^{m/r} = e$ Thus, $FV_t = PV(e)^{rt}$

Example: PV = 10000 r = 10% t = 5

$FV = 10000(e)^{.10(5)} = 10000\ e^{0.5} = 10000\ (1.6487) = 16487$

Finding PV: $PV = FV/\ (e)^{rt}$

If you want to have 200,000 at retirement 10 years from now and you can invest at 10% interest, how much should you invest today?

$PV = 200{,}000/\ (e)^{.10(10)} = 200000/e = 73578$

5-73

Computing APRs from EARs

$$APR = m\left[(1+EAR)^{1/m} - 1\right]$$

M = number of compounding periods per year

Suppose you want to earn an effective rate of 12% and you are looking at an account that compounds on a monthly basis. What APR must they pay?

$$APR = 12\left[(1+.12)^{1/12} - 1\right] = .1138655 \text{ or } 11.39\%$$

Excel Solution

=NOMINAL(0.12,12)

Effect rate	0.12	
Npery	12	
	NOMINAL=	0.113865515

ICONV: EFF = 12
C/Y = 12
NOM = 11.3866

5-74

Computing Payments with APRs

- Suppose you want to buy a new computer.
- The store is willing to allow you to make monthly payments.
- The entire computer system costs $3,500.
- The loan period is for 2 years.
- The interest rate is 16.9% with monthly compounding.
- What is your monthly payment?

=PMT(0.0140833,24,3500,0)

Excel Solution

Rate	.169/12	0.014083	
Nper		24	
PV		3500	
		PMT	($172.88)

5-75

Future Values with Monthly Compounding

Suppose you deposit $50 a month into an account that has an APR of 9%, based on monthly compounding. How much will you have in the account in 35 years?

420 N (35*12)
0.75 I/Y (9/12)
0 PV
-50 PMT
CPT FV = 147,089.22

Excel Solution

=FV(0.0075,420,-50,0)

Rate	.09/12	0.0075	
Nper		420	
Pmt		-50	
		FV =	$147,091.51

5-76

Present Value with Daily Compounding

You need $15,000 in 3 years for a new car. If you can deposit money into an account that pays an APR of 5.5% based on daily compounding, how much would you need to deposit?

Excel Solution

=PV(0.00015,1095,0,15000)

1095	N (3*365)
.015068493	I/Y (5.5/365)
0	PMT
15,000	FV
CPT PV = -12,718.56	

Rate	.055/365	0.000151		
Nper		1095		
FV		15000		
			PV =	($12,718.56)

5-77

Pure Discount Loans

- **Treasury bills are excellent examples of pure discount loans.**
  - **Principal amount is repaid at some future date**
  - **No periodic interest payments**
- **If a T-bill promises to repay $10,000 in 12 months and the market interest rate is 7 percent, how much will the bill sell for in the market?**

Excel Solution

PV(.07,1,0,10000)

1 N; 10,000 FV; 7 I/Y; CPT PV = -9345.79

Rate	0.07		
Nper	1		
FV	10000		
		PV=	($9,345.79)

5-78

Amortized Loan with Fixed Payment Example

- **Each payment covers the interest expense plus reduces principal**
- **Consider a 4-year loan with annual payments. The interest rate is 8% and the principal amount is $5000.**
  - **What is the annual payment?**

Excel Solution

= PMT(0.08,4,5000,0) = 1509.60

Rate	0.08		
Nper	4		
Pv	5000		
		PMT	($1,509.60)

5-79

Amortized Loan with Fixed Payment - Example

Year	Beginning Balance	Total Payment Payment	Interest Paid	Principal Paid	Ending Balance
1	$ 5,000.00	$ 1,509.60	$ 400.00	$ 1,109.60	$ 3,890.40
2	$ 3,890.40	$ 1,509.60	$ 311.23	$ 1,198.37	$ 2,692.03
3	$ 2,692.03	$ 1,509.60	$ 215.36	$ 1,294.24	$ 1,397.79
4	$ 1,397.79	$ 1,509.60	$ 111.82	$ 1,397.79	$ -
Totals		$ 6,038.40	$ 1,038.42	$ 5,000.00	

Interest Paid = Beginning Balance * Rate (8%)

Principal Paid = Total Payment – Interest Paid

Ending Balance = Beginning Balance – Principal Paid

5-80