Chapter 5 - Discrete Probability Distributions

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Random Variables

A random variable is a numerical description of the outcome of an experiment.

A discrete random variable may assume either a finite number of values or an infinite sequence of values.

A continuous random variable may assume any numerical value in an interval or collection of intervals.

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Random Variables…

A ***random variable*** is a function or rule that assigns a number to each outcome of an experiment. Basically it is just a symbol that represents the outcome of an experiment.

X = number of heads when the experiment is flipping a coin 20 times.

C = the daily change in a stock price.

R = the number of miles per gallon you get on your auto during the drive to your family's home.

Y = the amount of sugar in a mountain dew (not diet of course).

V = the speed of an auto registered on a radar detector used on I-83

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Two Types of Random Variables…

Discrete Random Variable – usually count data [Number of]

* one that takes on a ***countable*** number of values – this means you can sit down and list **all** possible outcomes without missing any, although it might take you an infinite amount of time.

X = values on the roll of two dice: X has to be either 2, 3, 4, …, or 12.

Y = number of customer at Starbucks during the day

Y has to be 0, 1, 2, 3, 4, 5, 6, 7, 8, ……………"real big number"

Continuous Random Variable – usually measurement data [time, weight, distance, etc]

* one that takes on an uncountable number of values – this means you can never list all possible outcomes even if you had an infinite amount of time.

X = time it takes you to walk home from class: X > 0, might be 5.1 minutes measured to the nearest tenth but in reality the actual time is 5.10000001…………………. minutes?)

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Random variables

Experiment	Random Variable (x)	Possible values for the random variable	Type
Inspect a shipment of 50 radios	Number of defective radios	0,1,2, 3 …….50	Discrete with finite values
Operate a restaurant for one day	Number of customers	0,1,2,3 ……	Discrete with infinite values
Fill a soft drink can (max = 12.1 ounces)	Number of ounces	$0 \le x \le 12.1$	Continuous
Operate a bank	Time between customer arrivals in minutes	$x \ge 0$	Continuous

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Discrete Probability Distributions

The probability distribution for a random variable describes how probabilities are distributed over the values of the random variable.

We can describe a discrete probability distribution with a table, graph, or formula.

The probability distribution is defined by a probability function, Denoted by $f(x)$, which provides the probability for each value of the random variable.

The required conditions for a discrete probability function are:

$f(x) \ge 0$

$\Sigma f(x) = 1$

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Discrete Uniform Probability Distribution

The discrete uniform probability distribution is the simplest example of a discrete probability distribution given by a formula.

The discrete uniform probability function is

$f(x) = 1/n$

the values of the random variable are equally likely

where:

n = the number of values the random variable may assume

Example would be tossing a coin (Head or Tails) and $f(x) = (1/n) = 1/2$ or rolling a die where $f(x) = (1/n) = 1/6$

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Discrete Probability Distributions

Example: DiCarlo Motors

Using past data on daily car sales for 300 days, a tabular representation of the probability distribution for sales was developed.

Number of cars sold	Number of days	x	$f(x)$
0	54	0	.18
1	117	1	.39
2	72	2	.24
3	42	3	.14
4	12	4	.04
5	3	5	.01
Total	300		1.00

Expected Value

The expected value, or mean, of a random variable is a measure of its central location.

$$E(x) = \mu = \Sigma x f(x)$$

The expected value is a weighted average of the values the random variable may assume. The weights are the probabilities.

The expected value does not have to be a value the random variable can assume.

Variance and Standard Deviation

The variance summarizes the variability in the values of a random variable.

$$\text{Var}(x) = \sigma^2 = \Sigma(x - \mu)^2 f(x)$$

The variance is a weighted average of the squared deviations of a random variable from its mean. The weights are the probabilities.

The standard deviation, σ, is defined as the positive square root of the variance.

Discrete Probability Distributions

Example: DiCarlo Motors

x	$f(x)$	$xf(x)$
0	.18	.00
1	.39	.39
2	.24	.48
3	.14	.42
4	.04	.16
5	.01	.05
	1.00	1.50

$E(x)$ = 1.50 = expected number of cars sold in a day

Discrete Probability Distributions

Example: DiCarlo Motors

x	$x - \mu$	$(x - \mu)^2$	$f(x)$	$(x - \mu)^2 f(x)$
0	0 – 1.5 = - 1.5	2.25	.18	2.25 (.18) = .4050
1	1 – 1.5 = -.5	.25	.39	.0975
2	2 – 1.5 = .5	.25	.24	.0600
3	3 – 1.5 = 1.5	2.25	.14	.3150
4	4 – 1.5 = 2.5	6.25	.04	.2500
5	5 – 1.5 = 3.5	12.25	.01	.1225
			1.00	1.2500

Variance of daily sales = σ^2 = 1.25

Standard deviation of daily sales = 1. 118 cars

Using Excel to compute the Expected value, Standard Deviation and Variance

- Excel Formula Worksheet

	A	B	C	D
1	Sales	Probability	Sq deviation from Mean	
2	0	0.18	=(A2-B9)^2	
3	1	0.39	=(A3-B9)^2	
4	2	0.24	=(A4-B9)^2	
5	3	0.14	=(A5-B9)^2	
6	4	0.04	=(A6-B9)^2	
7	5	0.01	=(A7-B9)^2	
8				
9	Mean	=SUMPRODUCT(A2:A7,B2:B7)		
10				
11	Variance	=SUMPRODUCT(C2:C7,B2:B7)		
12				
13	Std Deviation	=SQRT(B11)		

	A	B	C	D
1	Sales	Probability	Sq deviation from Mean	
2	0	0.18	2.25	
3	1	0.39	0.25	
4	2	0.24	0.25	
5	3	0.14	2.25	
6	4	0.04	6.25	
7	5	0.01	12.25	
8				
9	Mean	1.5		
10				
11	Variance	1.25		
12				
13	Std Deviation	1.118		