DETERMINING EQUILIBRIUM INCOME AND THE MULITIPLERS

We assume there is no foreign sector in the model or X = 0 and M = 0.

However, we will now add the government sector and include values for G and T (taxes).

The new aggregate expenditure or demand function is AE = C + I + G. In equilibrium

$$Y = C + I + G$$

SOLVING FOR EQUILIBRIUM INCOME

Now that all the components of aggregate demand are defined, the model can be solved for equilibrium income. Of course, the model will become more complicated as more variables are added.

Model 1 - No Government Sector

We will begin with the most basic model which assumes there is no government sector (G = 0, T = 0).

We let MPC = b

Thus, the consumption function becomes

$C = a + b(Y_D) = a + b\,(Y)$, since $= Y_D = Y - T = Y - 0 = Y$.

Aggregate Expenditure

$AE = a + bY + I$ At equilibrium: $Y = AE$ or $Y = a + bY + I$

Solving for Y:

$$Y - bY = a + I \quad \Rightarrow \quad Y\,(1 - b) = a + I$$

$$Y = \frac{a + I}{(1 - b)}$$

We can calculate the multiplier by the method of subtracting income in period 1 from income in period 0,

Define equilibrium income in time period 0 as:

$Y_0 = \dfrac{a_0 + I_0}{1 - b}$, where the subscript 0 denotes the values in time period 0.

The value of b or MPC is constant over time.

If the value of I is changed from time period 0 to 1, the new value of I is I_1. Holding all other values constant, the value of equilibrium income in time period 1 is:

$$Y_1 = \frac{a_0 + I_1}{1 - b}$$

Change in Y or $\Delta Y = Y_1 - Y_0 = \frac{a_0 + I_1}{1 - b} - \frac{a_0 + I_0}{1 - b} =$

$$\frac{a_0 + I_1 - a_0 - I_0}{1 - b} = \frac{I_1 - I_0}{1 - b} = \frac{\Delta I}{1 - b}$$

$\Delta Y = \alpha \Delta I$, where $\alpha = \frac{1}{1 - b}$

This is the simple investment multiplier.

The multiplier for autonomous consumption (a) can also be calculated in a similar fashion.

Model 2- Basic Macro Model with Government (Lump-sum taxes)

We continue to assume that there is no foreign sector in the model or X = 0 and M = 0.

However, we will now add the government sector and include values for G and T.

We will assume that taxes are lump-sum. Thus, T is constant for the period. .

The new aggregate expenditure function is AE = C + I + G. In equilibrium

$$Y = C + I + G$$

To solve for equilibrium, we need the following:

a) Consumption

$C = a + bY_D$
Y_D = disposable income = after-tax income
$Y_D = Y - T$

b) Investment and Government Spending

I = I (constant) G = G (constant)

Using the new equations we can now solve for the equilibrium level of income:

SOLVING FOR EQUILIBRIUM

$Y = C + I + G$
$Y = a + b(Y_D) + I + G$
$Y = a + b(Y - T) + I + G$
$Y = a + bY - bT + I + G$

$Y - bY = a - bT + I + G$
$Y(1 - b) = a - bT + I + G$

$$Y = \frac{a - bT + I + G}{1 - b}$$

Calculating the multiplier by the method above.

1. Multiplier for G

Define equilibrium income in time period 0 as:

$$Y_0 = \frac{a - bT_0 + I_0 + G_0}{1 - b}$$

The value of a and b or MPC is constant over time.

If the value of G is changed from time period 0 to 1, the new value of G is G_1. Holding all other values constant, the value of equilibrium income in time period 1 is:

$$Y_1 = \frac{a - bT_0 + I_0 + G_1}{1 - b}$$

$$Y_0 = \frac{a - bT_0 + I_0 + G_0}{1 - b}$$

Change in Y or $\Delta Y = Y_1 - Y_0 = \dfrac{a - bT_0 + I_0 + G_1}{1 - b} - \dfrac{a - bT_0 + I_0 + G_0}{1 - b}$

$$= \frac{a - bT_0 + I_0 + G_1 - a + bT_0 - I_0 - G_0}{1 - b} = \frac{G_1 - G_0}{1 - b}$$

Since $\Delta G = G_1 - G_0$

$\Delta Y = \alpha \, \Delta G$, where $\alpha = \dfrac{1}{1 - b}$

This is the simple multiplier for Government Spending.

2. Multiplier for T

Define equilibrium income in time period 0 as:

$$Y_0 = \frac{a - bT_0 + I_0 + G_0}{1 - b}$$

The value of a and b or MPC are constant over time.

If the value of T is changed from time period 0 to 1, the new value of T is T_1. Holding all other values constant, the value of equilibrium income in time period 1 is:

$$Y_1 = \frac{a - bT_1 + I_0 + G_0}{1 - b}$$

$$Y_0 = \frac{a - bT_0 + I_0 + G_0}{1 - b}$$

Change in Y or $\Delta Y = Y_1 - Y_0 = \frac{a - bT_1 + I_0 + G_0}{1 - b} - \frac{a - bT_0 + I_0 + G_0}{1 - b}$

$$= \frac{a - bT_1 + I_0 + G_0 - a + bT_0 - I_0 - G_0}{1 - b} = \frac{-bT_1 + bT_0}{1 - b}$$

$$= \frac{-b(T_1 - T_0)}{1 - b}$$

Since $\Delta T = T_1 - T_0$

$\Delta Y = \alpha_T \Delta T$, where $\alpha_T = \frac{-b}{1 - b}$

This is the simple multiplier for Lump-sum taxes. Notice that the multiplier is smaller that the multiplier for government spending and opposite in sign.

Model 3 (Lump-sum Taxes and Transfer Payments):

a = 100, b = MPC = 0.75, I = 100, G = 200, TR = 100,
T = 200

1) Calculate equilibrium Y.

$Y = C + I + G$
$Y = a + b(Y_D) + I + G$
$Y = a + b(Y - T + TR) + I + G$
$Y = a + b\ (Y - T + TR) + I + G$
$Y = a + bY - bT + bTR) + I + G$
$Y - bY = a - bT + bTR + I + G$
$Y(1 - b) = a - bT + bTR + I + G$

$$Y = \frac{a - bT + bTR + I + G}{(1 - b)}$$

Substituting the values given for each variable yields:

$$Y = \frac{100 - (0.75)(200) + (0.75)(100) + 100 + 200}{1 - .75} = \frac{325}{0.25} = 1300$$