SOLUTIONS
EC302 - INTERMEDIATE MICROECONOMICS
Loyola University
Fall 2016

Problem Set 3

1. A steel plant's production function is $Q = 0.025LK$, where Q is the daily output rate, L is the number of workers it uses per day, and K is the unit of daily capital employment. According to this production function, the marginal products of capital and labor are, respectively,

$$MP_K = 0.025L \qquad MP_L = 0.025K$$

Suppose that the price of labor is $100 per worker per day, and that the price of capital is $200 per unit per day.

a. The firm's vice president for manufacturing hires you to figure out what combination of inputs the plant should use to produce 20 units of output per day. What advice would you give?

b. How much would it cost per day to produce this level of output?

c. Now suppose that the firm increases this plant's budget to $120,000 a day and instructs the plant manager to produce as much steel as possible while not exceeding this budget. Again your advice is sought. How much capital and labor would you advise employing? How much output should the firm expect from this plant?

Solution

a) $MP_L/MP_K = w/r \Leftrightarrow .025K/.025L = 100/200$ or $K/L = .5$ or $L = 2K$

$Q = f(K,L) \Leftrightarrow 20 = .025LK$ Substituting $L = 2K$ into above function, we have

$20 = 0.025(2K)(K) = 0.025(2K^2) \Leftrightarrow 20 = .05K^2 \Leftrightarrow 2000/5 = K^2 \Leftrightarrow K^* = 20$. Hence, $L^* = 40$.

b) $200(20) + 40(100) = 4000 + 4000 = 8000$/day

c) 120000 = K(200) + L(100). But we know that we must use the ratio 2K = L to produce with the optimal input combination. Hence, we have 120000 = K(200) + 2K(100) or K = 120000/400 or K**=300, L** = 600. At this level of input use, we can produce .025(300)(600) = 25(180) = 4500.

2. Suppose you are assured by the owner of a plant that his plant is subject to constant returns to scale, with labor and capital the only inputs. He claims that output per worker in his plant is a function of capital per worker only. Is he right? Prove.

Solution $Q = K^{\alpha} L^{1-\alpha}$ $\frac{Q}{L} = K^{\alpha} L^{1-\alpha} L^{-1} = K^{\alpha} L^{-\alpha} = (\frac{K}{L})^{\alpha}$ He is right.

3. Do the following production functions exhibit decreasing, constant or increasing returns to scale?

a. Q = 0.5KL b. Q = 2K + 3L

Solution

a. Q = 0.5KL

Therefore, we can substitute m*K* for *K* and m*L* for *L*, and check the result against an equal increase in *Q*.

$Q^* = 0.5(mK)(mL) = m^2(0.5KL) = m^2Q$

t >1 This production function exhibits increasing returns to scale.

b. Q = 2K + 3L $Q^* = 2(mK) + 3(mL) = m(2K + 3L) = mQ$.

t = 1 This production function exhibits constant returns to scale.

4. The production function for the personal computers of DISK, Inc., is given by

$Q = 10K^{0.5}L^{0.5}$, where Q is the number of computers produced per day, K is hours of Machine time, and L is hours of labor input. DISK's competitor, FLOPPY, Inc., is using the production function $Q = 10K^{0.6}L^{0.4}$.

If both companies use the same equal amounts of capital and labor, which firm will generate more output?

Solution

Let Q be the output of DISK, Inc., Q_2, be the output of FLOPPY, Inc., and X be the same equal amounts of capital and labor for the two firms. Then, according to their production functions,

$Q = 10X^{0.5}X^{0.5} = 10X^{(0.5 + 0.5)} = 10X$ *and* $Q_2 = 10X^{0.6}X^{0.4} = 10X^{(0.6 + 0.4)} = 10X$.

Because $Q = Q_2$, both firms generate the same output with the same inputs.

Note: If the two firms both used the same amount of capital and the same amount of labor, but the amount of capital was not equal to the amount of labor, then the two firms would not produce the same level of output. In fact, if $K>L$ then $Q_2>Q$.

5. A firm's production function for footballs is given by $q = 4L^{1/2}K^{1/2}$.

Answer the following questions with regard to the firm’s output and costs. If the price of capital is 100, the price of labor is 1, and the firm has 4 units of capital, derive the firm's total cost, fixed cost, variable cost, average total cost, average fixed cost, average variable cost, and marginal cost functions. (Remember, these cost functions express cost as a function of output, q, not labor).

Solution

$TC = TFC + TVC = rK + wL = 100(4) + 1(L) = 400 + L$. But TC must be expressed in terms of q rather than L. From the production function, $q = 4L^{1/2}4^{1/2} = 8L^{1/2}$ => $L^{1/2} = q/8$ => $L = q^2/64$. Substituting this into the total cost function yields

$\underline{TC = 400 + q^2/64}$.

Therefore, $\underline{ATC = 400/q + q/64}$, $\underline{AVC = q/64}$, $\underline{AFC = 400/q}$, and $\underline{MC = q/32}$. Note that AVC and MC are straight lines emanating from the origin. Both are upward sloping and MC > AVC for all $q \geq 0$.