Solutions
EC302 - INTERMEDIATE MICROECONOMICS
Loyola University
Fall 2017

Problem Set 2

1. Draw the indifference curves for the following individual's preferences for spring rolls and noodles.

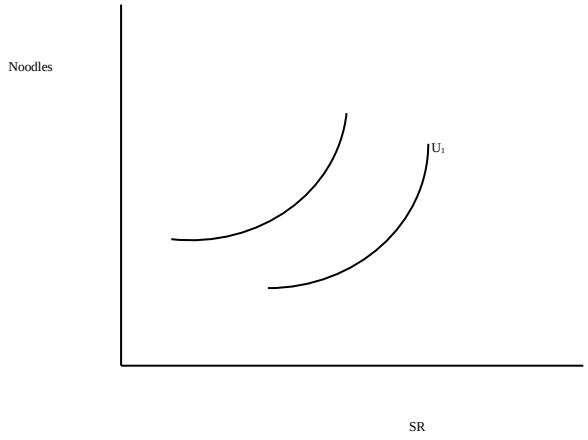
a) Ms. H loves spring rolls, but hates noodles. She always prefers more spring rolls no matter how much noodles she has.

b) Mr. D is indifferent between bundles of either three spring rolls or two noodles. His preferences do not change as she consumes more of either good.

c) Ms. G eats one spring rolls and washes it down with one noodles. She will not consume an additional order of spring rolls without one more noodle.

Solution

a)



b) These two goods are perfect substitutes in consumption. The indifference curve are parallel lines with slope of -2/3 (assuming noodles on the Y-axis). Look at the notes to see the indifference curves for perfect substitutes.

c) These two goods are perfect complements and there is one indifference curve.

2. Show that the two utility functions given below generate the identical demand functions for goods x and y:

a. U(x,y) = log(x) + log (y) b. U(x,y) = $(xy)^{0.5}$

Solution

We know that an individual will maximize utility by consuming where

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_y}$$

You can also show the above result by using the method of Lagrangian multipliers to find the actual demand functions.

a). Notice the function is a Cobb-Douglas function that can also be written as

U(xy) = xy. Thus, $Y/X = P_x/P_y$

Looking at the Budget Constraint M) = 2PxX or $M = 2P_YY$

Demand curve for X is X = M/2Px and Demand curve for Y = M/2Py

b) The Lagrangian is,

$$L = X^{0.5}, Y^{0.5} + \lambda(M - P_X X - P_Y Y)$$

Short cut: $P_x/P_y = MU_x/MU_y = Y/X$ (since $MU_x = MU_X = 0.5X^{-0.5}Y^{0.5}$ and $MU_Y = 0.5X^{0.5}Y^{-0.5}$)

so $P_x X = P_y Y$

Looking at the Budget Constraint $M = 2P_xX$

Demand curve for x is $X = M/2P_x$ and Demand curve for Y = M/2Py

This is also what you would find if you applied the same procedure to the other utility function.

3. Consider the following Utility function. $U = X^{\alpha}Y^{\beta}$

a. Compute the MU_X and MU_Y

b. Prove that if $0 < \alpha < 1 and\ 0 < \beta < 1$

the marginal utility of both X and Y are subject to diminishing marginal utility.

Solution

a. Compute the MU_X and MU_Y

$$MU_X = \frac{\partial U}{\partial X} = \alpha X^{\alpha-1}Y^{\beta} \quad MU_Y = \frac{\partial U}{\partial Y} = \beta X^{\alpha}Y^{\beta-1}$$

b. Prove that if $0 < \alpha < 1 and\ 0 < \beta < 1$ the marginal utility of both X and Y are subject to diminishing marginal utility.
You can see that if $\alpha\ or\ \beta < 1$ we diminishing marginal utility.

$$\frac{\partial MU_X}{\partial X} = \frac{\partial^2 U}{\partial X^2} = (\alpha - 1)\alpha X^{\alpha-2}Y^{\beta} < 0$$
$$\frac{\partial MU_Y}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} = (\beta - 1)\beta X^{\alpha}Y^{\beta-2} < 0$$

4. Suppose the demand for good X is: $Q_x = 500P_x^{-1}I^{0.5}$. Compute both price and income elasticity of demand. Show work.

Solution $E_d = 1$ $E_I = 0.5$

5. Analysis indicates that if cocaine (or heroin) prices increase 5%, the quantity demanded will decrease by 1%. Answer the following questions with regard to the market for cocaine and anti-cocaine policies.

a. What elasticity of demand is implied by this analysis?

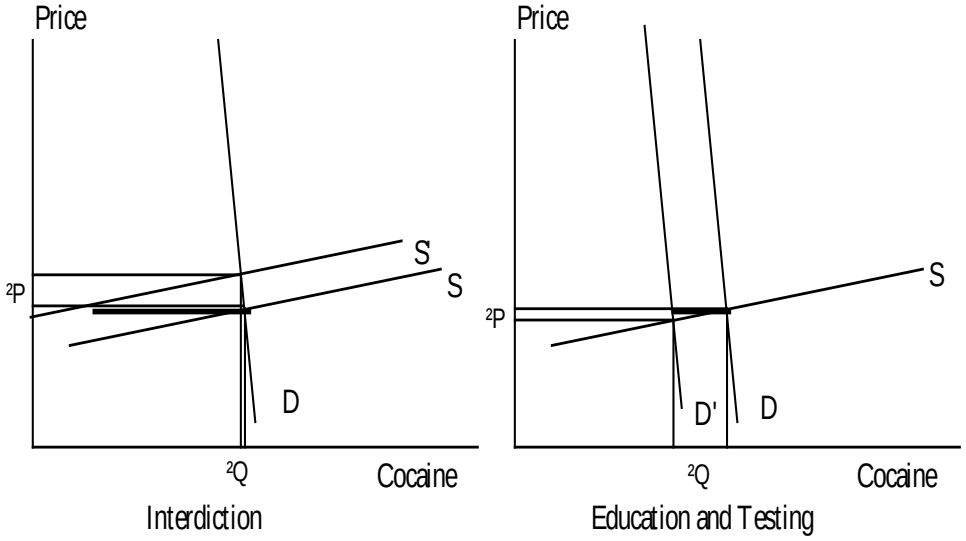
b. If the supply for cocaine is relatively elastic, assess the impact on cocaine prices and the quantity exchanged for the following two anti-cocaine policies. Use graphs to illustrate your answers as appropriate.
   i. Drug interdiction (ban on drugs, for example) efforts to reduce the supply of drugs.
   ii. Drug education and testing programs to reduce the demand for cocaine.

c. Which policy would be most effective in reducing cocaine use? Why?

Solution

a. $E_d = \%\Delta Q / \%\Delta P = 1\%/5\% = -0.2$ or $|-0.2| = 0.2$

b/c. If supply is relatively elastic and demand is relatively inelastic in the current region of price and quantity, the demand curve is relatively steep and the supply curve relatively flat. In this case shifts in the supply (interdiction) and demand (education and testing) will have different effects on equilibrium price and quantity (see figures).



With an elastic supply and inelastic demand, shifts in the supply curve will have little impact on equilibrium quantity, while they will raise equilibrium price. As supply is reduced, most of this decrease will be realized through price increases, because demand is inelastic. Furthermore, small price increases will cause large increases in the quantity supplied (movement along the new supply curve). With curves shaped as above, interdiction will increase price without significantly decreasing quantity consumed. Alternatively, decreases in demand will primarily decrease equilibrium quantity without significantly reducing equilibrium price. As demand decreases, prices will tend to fall. With elastic supply, this decrease in price will cause a large decrease in the quantity supplied. With inelastic demand, the decrease in price will not significantly increase quantity demanded. Thus, education and testing are likely to have a larger impact on equilibrium quantity, but only a small affect on price.

6. The demand curve for beer is $Q_B = 50000 - 500\,P_B$ where Q_B is the quantity demanded of beer (in bottles) and P_B is the price of beer (in $ per bottle).

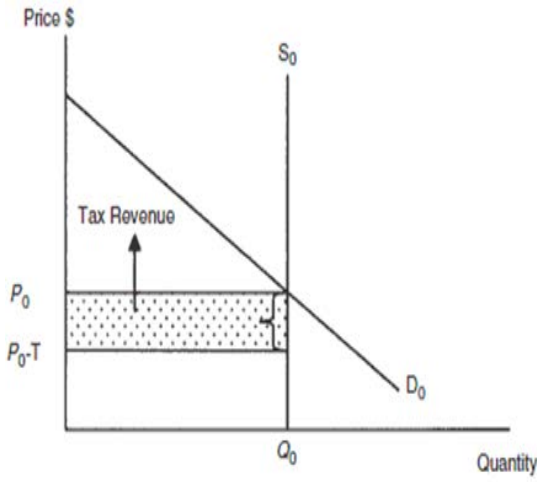
a. If the supply of beer is perfectly inelastic at a quantity of 40000 bottles per year, what is the equilibrium price of a bottle of beer?

b. What would be the effect on the price of a bottle of beer if the government imposes a tax of 5 cents per bottle? (Show graphically and explain)

Solution

a. Solving for P $\quad 50000 - 500P_B = 40000 \quad 500P_B = 10000 \quad P_B = 20$

b. What would be the effect on the price of a bottle of beer if the government imposes a tax of 5 cents per bottle? (Show graphically and explain) Nothing



7. The price consumption curve showing the effect of an increase in the price of X is upward sloping. Given this information is the value of the elasticity of good X elastic or inelastic. Prove both mathematically and graphically.

Solution

Look at the Budget Constraint. $M = P_X X + P_Y Y$

Take the total derivative. $dM = P_X dX + X dP_X + P_Y dY + Y dP_Y$

Since $dM = dP_Y = 0 \qquad P_X dX + X dP_X + P_Y dY = 0$

Solve for the elasticity of X.

$$\frac{P_X dX}{XdP_X}+\frac{XdP_X}{XdP_X}+\frac{PydY}{XdP_X}=0 \qquad -E_x=\left|E_x\right|=1+\frac{PydY}{XdP_X}$$

if P_X rises or $dP_X > 0$, and $dY < 0$, which would occur if the price consumption curve is upward sloping, then $\left|E_x\right|<1$