CHAPTER 4 LECTURE - CONSUMER CHOICE

Our objective is to construct a simple model of consumer behavior that will permit us to predict consumers' reactions to changes in their opportunities and constraints. We will take tastes and preferences as given, but we will represent them with a very general analytical model.

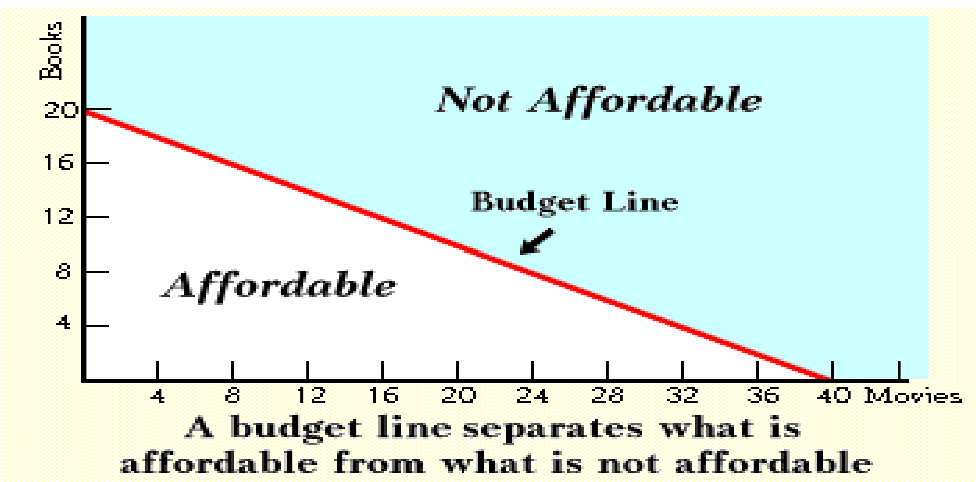
We will Now Look at the Consumer's Opportunity Set or Budget Constraint

General Formulation: Assume for the moment there are only two goods in the world -- this is a simplification we will relax later -- a person therefore will spend all his income on these two goods.

This can be depicted as $I = P_X X + P_Y Y$, where the first term is the person's total expenditure on X and the second term the total expenditure on Y.

Some texts use M for Income.

Let's graph this using some numbers, suppose I = \$200, P_X = \$5, and P_Y = \$10 -- note, as is usually the case, this person is a price taker. From this information we can graph a budget constraint.



What is the maximum amount of X (Movies) this person can buy? 40 units. How does one know that? From the general formula, (Income or I)/P_X .

a) What is the maximum amount of Y this person can buy? 20 units.
b) Suppose she wants one unit of X -- now what is the maximum amount of Y she could purchase? It would be 18 -- so when you buy ONE X, you give up TWO Y -- this is the REAL or RELATIVE price of X.

Of course, we can put this into a general formula, as follows:

$$I = P_X X + P_Y Y \qquad P_Y Y = I - P_X X \quad \text{or} \quad Y = \frac{I}{P_Y} - \left(\frac{P_X}{P_Y}\right) X$$

The first term is the intercept of the Y-axis; the second term is the slope. Note **the slope is the price of X over the price of Y -- the relative price or** $-\frac{P_X}{P_Y}$**.**

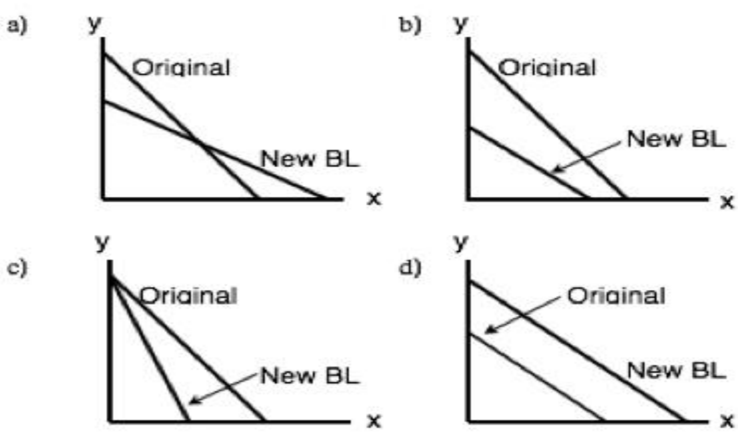
We noted that anything within (below and to the left) of the budget line is obtainable; anything beyond the budget line is not obtainable. In this world where we are spending all our income, we will always be on the budget line.

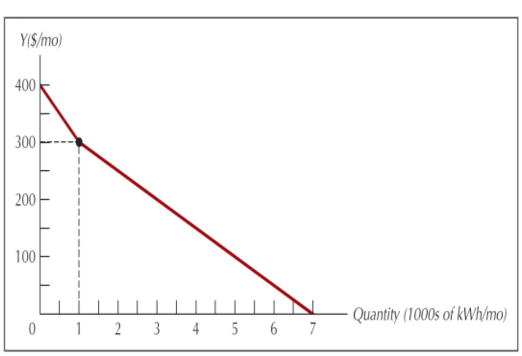
Changes in the Budget Constraint.

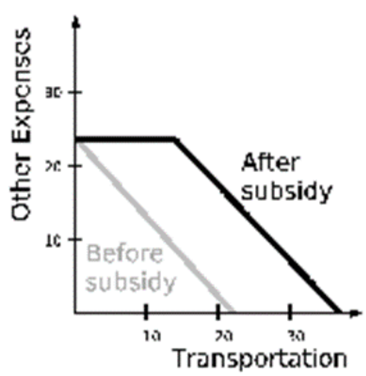
There are three givens when we construct the budget line, income, price of X, and price of Y -- if any of these things change, the budget line changes. What happens if:

a) **Income increases?**
b) **Price of X goes up?**
c) **Price of Y goes down?**
d) **Given a gift certificate of $20 for X?**

What has happened here?



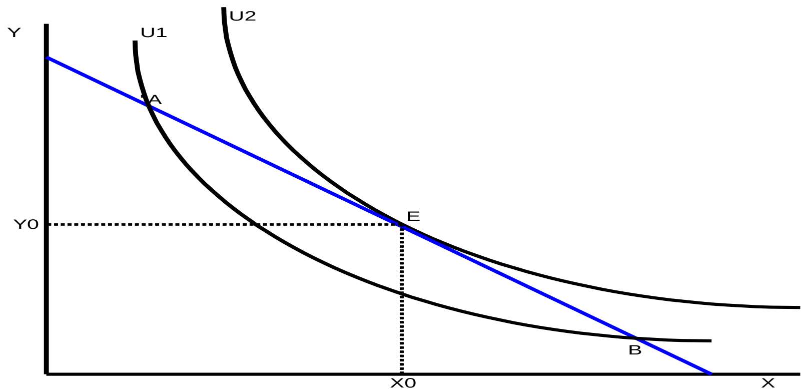




Consumer Equilibrium

Now it's time to bring these two concepts together and build a model of human behavior. Look at the following graph:

We want to Maximize Utility, U(x,y), subject to our constraint $P_X X + P_Y Y \leq I$



At point A, what is the slope of the indifference curve? Remember the slope is the marginal rate of substitution that tells us what the **maximum amount of Y the person is willing to give up to obtain one more unit of X** -- let's say the slope is 10.

At point A, what is the slope of the budget constraint? Remember, the slope is the relative price of X -- in other words, **what do I have to pay**? Let's say it is 2 -- which means that to obtain one more unit of X, I give up two units of Y.

Now, if I am willing to give up 10 but only have to give up 2, what will happen to my utility -- clearly I am better off and therefore, by definition, I will move up a higher indifference curve.

So if we move up a higher indifference curve, we go through the same decision process -- **is what I'm willing to pay greater than what I have to pay?** -- if so, I buy it and am better off as a result -- this is identical to what we did with a demand curve -- in fact, later this week we will derive a demand curve from this graph.

Now look at point E -- at this point, the indifference curve is tangent to the budget line. At the point of tangency, what can you say about the slopes? By definition, they are equal. So what does that say about this person? Well the maximum amount I am willing to pay is equal to the amount I have to pay -- i.e. MRS is to the price ratio.

If I go beyond that, then the maximum amount I'm willing to pay is less than what I have to pay and therefore, if I purchase more, I make myself worse off.

So point E gives us is one point on the demand curve. For a given income, price of X, and price of Y, and given my preferences this is how much X (and Y) I will consume.

So E is the equilibrium point -- that is where we will always be (or get to very quickly).

Remember we are doing comparative statics -- which is just the comparison of different equilibrium points.

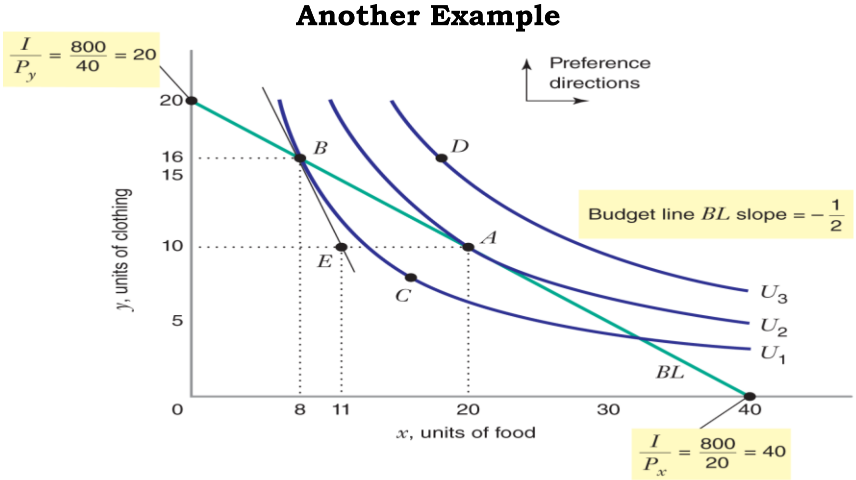
Notice that At point E, the slopes are equal. We know the slope of the indifference curve is the marginal rate of substitution:

$MRS = -\frac{MU_X}{MU_Y}$ The slope of the budget constraint is the price ratio: $-\frac{P_X}{P_Y}$.

Therefore, at point E the following is true: $\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$ or $\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$

The economic interpretation of this is that at equilibrium the utility per dollar spent must be equal across all goods. In other words, if I can take a dollar away from Y and spend it on X and get more utility, I will do that (and vice versa).

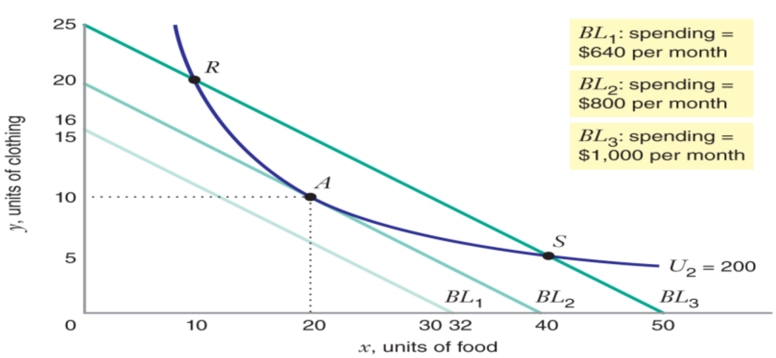
Another Example



Explain why Point B in the above diagram is NOT OPTIMAL

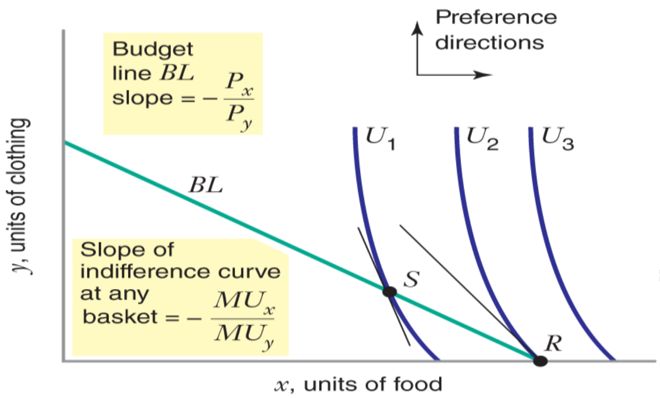
We can also look at this as an Expenditure Minimization Problem.

Minimize (x,y) expenditure = P_XX+P_YY subject to constraint that U(x,y)=U*

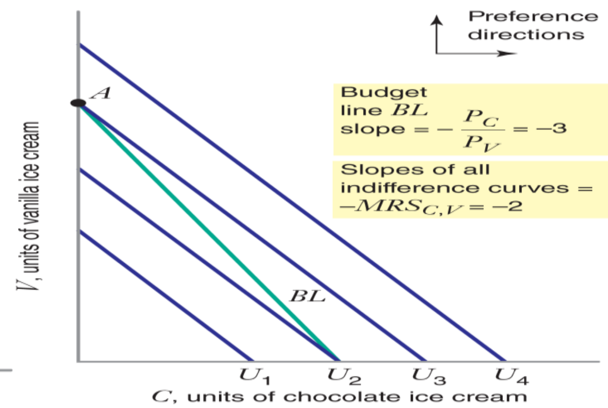


Some Special Cases

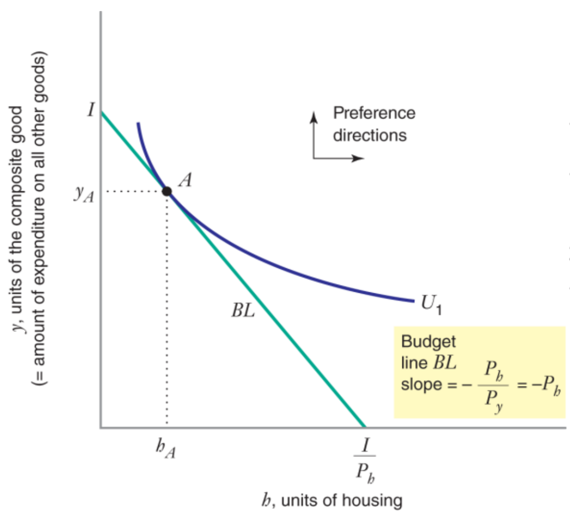
(Corner Solution)



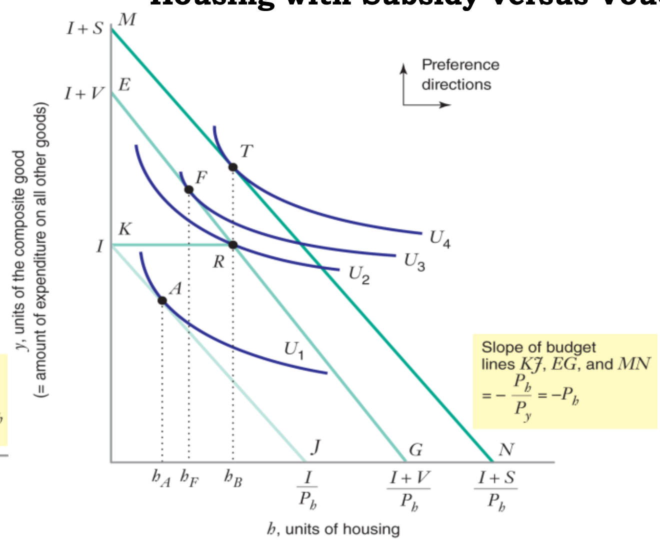
(Equilibrium with Perfect Substitutes)



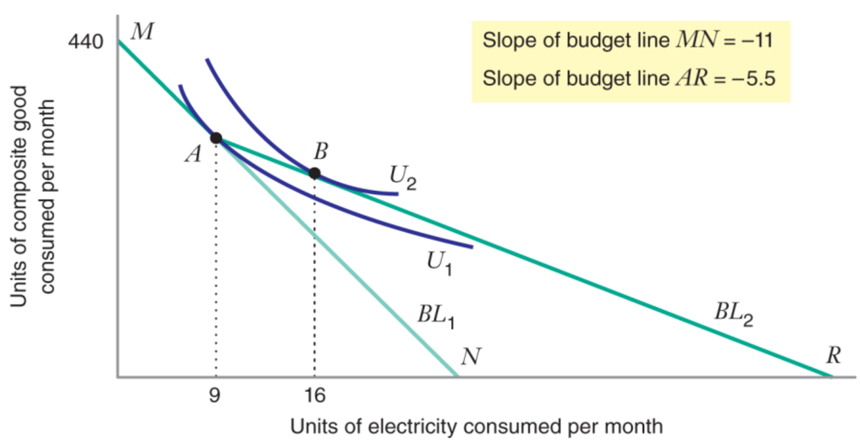
Consumer Choice with Composite Goods



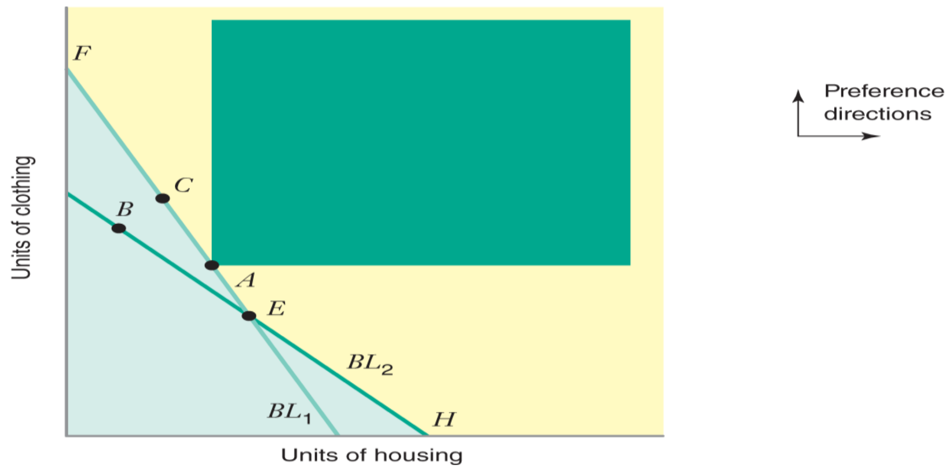
Housing with Subsidy versus Voucher



Quantity Discounts



REVEALED PREFERENCE



Suppose we do not know the consumer's indifference map, but we do have observations about consumer choice with two different budget lines. When the budget line is BL_1, the consumer chooses basket A. When the budget line is BL_2, the consumer chooses basket B. What does the consumer's behavior reveal about his preferences? As shown by the analysis in the text, the consumer's indifference curve through A must pass somewhere through the yellow area, perhaps including other baskets on EF.

First, the consumer chooses basket A when he could afford any other basket on or inside BL_1, such as basket B. Therefore, A is at least as preferred as B (A preferred to B). But he has revealed even more about how he ranks A and B. Consider basket C. Since the consumer chooses A when he can afford C, we know that A is preferred to C. And since C lies to the northeast of B, C must be strongly preferred to B ($C \succ B$). Then, by transitivity, A must be strongly preferred to B (if A is preferred to C and $C \succ B$, then $A \succ B$)

MATHEMATICAL ANALYSIS OF CONSUMER BEHAVIOR

1. The Indifference Curve

The total utility function is given as: *U(X, Y).*

We have an indifference curve that is given by: *U(X, Y) = C,*

where "C" is a constant level of utility along the stable indifference curve.

Take the total differential of this indifference curve and set it equal to zero (Why?): We call this the F.O.C. or first order condition.

U = U(X, Y) $$dU = \frac{\partial U}{\partial X} dX + \frac{\partial U}{\partial Y} dY = 0 \quad \text{or} \quad \frac{\partial U}{\partial X} dX + \frac{\partial U}{\partial Y} dY = 0$$

(**Note**: By taking the total differential of an indifference curve and setting it equal to zero, both *X* and *Y* are allowed to change simultaneously, but there is no change in total utility.)

We note that some texts define the MRS as the negative of the slope. If that is the case, solving for the negative of the slope of the indifference curve, *-dY/dX*:

$$-\frac{dY}{dX} = MRS_{X,Y} = \frac{\frac{\partial U}{\partial X}}{\frac{\partial U}{\partial Y}} \quad \text{or} \quad MRS_{X,Y} = \frac{MU_X}{MU_Y}$$

The slope of an indifference curve is the $MRS_{X,Y}$; which is equal to the ratio of the marginal utility of the two goods consumed. $MRS_{X,Y}$ should be positive if both MU_x and MU_y are positive.

2. The Budget Line

Budget Line (or **Income Constraint**)

The locus of budgets (total expenditures on alternative combinations of goods) that can be purchased if all money income is spent. Algebraically:

$I = P_x X + P_y Y$,

Where I is total money income, P_X is the nominal price of good *X*, P_Y is the nominal price of good *Y*, and *X* and *Y* are the quantities of both goods purchased.

Solving for Y: $Y = \frac{I}{P_Y} - \frac{P_X}{P_Y} X$ (**Note**: The slope of the budget line is the ratios of the price of X to Y.) Slope of Budget Line = $-\frac{P_X}{P_Y}$

If I = *$200*, P_X is *$20*, and P_Y is *$10*, then the budget line or income constraint is:

$$Y = \frac{\$200}{\$10} - \frac{\$20}{\$10} X \quad \text{or} \quad Y = 20 - 2X \quad \text{Thus:} \quad -\frac{P_X}{P_Y} = -\frac{20}{10} = -2$$

We often ignore the negative sign.

A consumer maximizes his/her total utility from consumption subject to the budget constraint.

Consumer equilibrium implies:

1. The consumer's highest indifference curve is tangent to the budget line. (Why?)
2. Equality between the $MRS_{X,Y}$ and the relative price ratio of good *X*. (Why?)
3. The ratios of the *MU* to the price are equal for all goods. (Why?)

(Consumer allocates expenditures so that the utility of the last dollar spent on each good is equal.)

The point of consumer equilibrium is where: $MRS_{XforY} = \frac{P_X}{P_Y}$ or $\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$

Cross-multiplying: $\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$

Formal Mathematical Proof of Consumer Equilibrium

Maximize: $U = U(X, Y)$ Subject to: $P_x X + P_y Y = I$,

(**Note**: A consumer maximizes his/her total utility given the budget constraint.)

This is a constrained maximization problem, hence, the Lagrangean function is:

$$£ = U(X,Y) - \lambda(P_X X + P_Y Y - I) = 0$$

Taking the derivative of £ with respect to each good (i.e., *X* and *Y*) and setting it equal to zero:

$$\frac{\partial £}{\partial X} = \frac{\partial U}{\partial X} - \lambda P_X = 0$$

$$\frac{\partial £}{\partial Y} = \frac{\partial U}{\partial Y} - \lambda P_Y = 0$$

$$\frac{\partial £}{\partial \lambda} = P_X X + P_Y Y - I = 0$$

Solving for λ: $\lambda = \frac{MU_X}{P_X}$ and $\lambda = \frac{MU_Y}{P_Y}$

Equating these two equations: $\frac{MU_X}{P_X} = \frac{MU_Y}{P_y}$

Consumer equilibrium requires that the ratio of the marginal utility and the money price of each good be equalized for all goods consumed.

Alternatively stated: $\frac{MU_X}{MU_Y} = \frac{P_X}{P_y}$

Consumer equilibrium requires that the ratio of the marginal utilities of the goods equal the relative price ratio.

NOTE: The negative sign in front of λ is arbitrary; a positive sign works equally well.

In the case we have $£ = U(X,Y) + \lambda(I - P_X X - P_Y Y) = 0$

Examples

Example 1 - Julie's preferences for food (measured by *F*) and clothing (measured by *C*) are described by the utility function *U(F,C) = FC.* Suppose food costs $1 per unit and clothing costs $2 per unit. She has an income of $12 to spend on food and clothing.

For this problem the Lagrangian function is:

$£ = FC - \lambda(P_F F + P_C C - I) = FC - \lambda(F + 2C - I)$

Solving:

$$MU_F - \lambda P_F = 0 \qquad => \; C - \lambda = 0$$

$$MU_C - \lambda P_C = 0 \qquad => \; F - 2\lambda = 0$$

$$P_F F + P_C C - I = 0 \quad > F + 2C - 12 = 0$$

Her marginal utilities are $MU_F = C$ and $MU_C = F$. Thus, both F and C are subject to diminishing marginal utility.

We have three equations and three unknowns. When we combine the first two equations, we find that Julie should equate the marginal utility per dollar spent on each good.

$$\lambda = \frac{MU_F}{P_F} = \frac{MU_C}{P_C} \quad => \lambda = \frac{C}{1} = \frac{F}{2} \quad and \;\; F = 2C\,.$$

Substituting $F = 2C$ into the budget constraint, we find that

$$F + 2C - 12 = 0 \quad => \; 2C \mp C - 12 = 0 \quad 4C = 12 \; > C = 3 \text{ and } F = 6\,.$$

The meaning of λ. We found that $\lambda = \frac{C}{1} = \frac{F}{2} \quad => \lambda = \frac{3}{1} = \frac{6}{2} = 3.$ This is the rate of change $\frac{\Delta U}{\Delta I}$ when Julie's income is exactly 12. If her income were to rise by a dollar, we would expect to see her utility increase by about 3.

When Julie's income is 12, she chooses $C = 3$ and $F = 6$. Her utility is $U = FC = (6)(3) = 18$. When her income is 13, she chooses $F = 6.5$ and $C = 3.25$. Her utility is $U = FC = (6.5)(4.25) = 21.25$.

Julie's utility increased by 3.25 (from 18 to 21.25) when her income increased by 1 (from 12 to 13). This is close to the values of λ we found in parts (c) and (d), as we expected.

Example 2 - The utility that Ann receives from consuming food (measured by *F*) and clothing (measured by *C*) is described by the utility function $U(F,C) = FC + F$. Suppose food costs $1 per unit and clothing costs $2 per unit. She has an income of $22 to spend on food and clothing.

We can see that Ann has a diminishing marginal rate of substitution of food for clothing. The marginal utilities for the two goods are $MU_F = C+1$ and $MU_C = F$. Since both marginal utilities are positive, we know that the indifference curves are negatively sloped.

When we increase F along an indifference curve, the level of C must therefore fall. We know that $MRS_{F,C} = \frac{MU_F}{MU_C} = \frac{C+1}{F}$. As we increase F along an indifference curve (and C falls), the value of $MRS_{F,C}$ falls. Therefore, we do have diminishing $MRS_{F,C}$.

This is important because it guarantees that the solution, we find using the method of Lagrange will maximize utility while satisfying the budget constraint.

For this problem the Lagrangian function is:

$$£ = (FC + F) + \lambda(I - P_F F - P_C C) = (FC + F) + \lambda(22 - F - 2C)$$

We set it up with a positive in front of λ just for fun.

$$MU_F - \lambda P_F = 0 \quad => \quad C + 1 - \lambda = 0$$
$$MU_C - \lambda P_C = 0 \quad => \quad F - 2\lambda = 0$$
$$I - P_F F - P_C C = 0 \quad => 22 - F - 2C = 0$$

We have three equations and three unknowns. When we combine the first two equations, we find that Ann should equate the marginal utility per dollar spent on each good.

$$\lambda = \frac{MU_F}{P_F} = \frac{MU_C}{P_C} \quad => \lambda = \frac{C+1}{1} = \frac{F}{2} \quad and \quad F = 2C + 2\,.$$

Substituting $F = 2C + 2$ into the budget constraint, we find that

$$22 - F - 2C = 0 \quad => \; 22 - 2C - 2 - 2C = 0 \quad => 4C = 20 \; or \; C = 5 \text{ and } F = 12\,.$$

Example 3 - Omar consumes only two goods, whose quantities are measured by *x* and *y*. His preferences are described by the utility function $U(x,y) = xy + 10(x + y)$. The prices of the goods are P_X = \$9 and P_y = \$3. He has a daily income of \$30.

The marginal utilities for the two goods are $MU_x = y+10$ and $MU_y = x+10$. Since both marginal utilities are positive, we know that the indifference curves are negatively sloped.

When we increase x along an indifference curve, the level of y must therefore fall.

We know that $MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{y+10}{x+10}$. As we increase x along an indifference curve (and y falls), the value of $MRS_{x,y}$ falls. Therefore, we do have diminishing $MRS_{x,y}$.

For this problem the Lagrangian function is:

$$£ = xy + 10(x+y) - \lambda\left(P_x x + P_y y - I\right) = xy + 10(x+y) - \lambda(9x + 3y - 30)$$

$$\begin{array}{lll} MU_x - \lambda P_x = 0 & => y + 10 - 9\lambda = 0 & x > 0 \\ MU_y - \lambda P_y = 0 & => x + 10 - 3\lambda = 0 & y > 0 \\ P_x x + P_y y - I = 0 & => 9x + 3y - 30 = 0 & \lambda > 0 \end{array}$$

We now have three equations and three unknowns. If Omar can do so, he should equate the marginal utility per dollar spent on each good. Combining the first two equations, we see that equating the marginal utility per dollar spent would require that:

$$\lambda = \frac{MU_x}{P_x} = \frac{MU_y}{P_y} \quad => \lambda = \frac{y+10}{9} = \frac{x+10}{3} \quad and \quad y = 3x + 20.$$

Substituting $y = 3x + 20$ into the budget constraint, we find that

$$30 - 9x - 3y = 0 \quad => 30 - 9x - 3(3x + 20) = 0 \quad => x = -\frac{5}{3} \text{ and } y = 15 .$$

But x cannot be negative so the utility-maximizing basket must be at a corner point, with either $x = 0$ or $y = 0$.

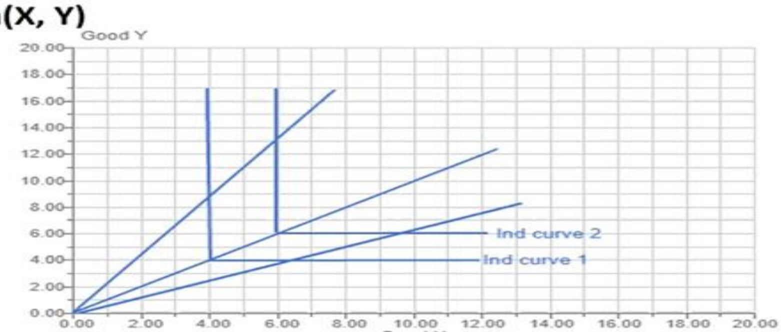
Homothetic Preferences - If the *MRS* depends only on the ratio of the amounts of the two goods, not on the quantities of the goods, the utility function is homothetic

We can show using the Cobb – Douglas function. $U = X^{\alpha} Y^{\beta}$

$$MRS = \frac{\frac{\partial U}{\partial X}}{\frac{\partial U}{\partial y Y}} = \frac{\alpha X^{\alpha-1} Y^{\beta}}{\beta X^{\alpha} Y^{\beta-1}} = \frac{\alpha}{\beta} \cdot \frac{Y}{X}$$

Homothetic Utility Functions:

- **Perfect substitutes: U = 6X + 3Y, MRS = 6/3 = 2**
- **U = XY, MRS = Y/X, so any Cobb-Douglas utility function**
- **Perfect complement: U = min(X, Y)**



Practice Problems

1. An individual consumes products X and Y and spends $24 per time period. The prices of the two goods are $3 per unit for X and $2 per unit for Y. The consumer in this case has a utility function expressed as:

$$U = X^{1/2}Y^{1/2}$$

   a. Determine the values of X and Y that will maximize utility in the consumption of X and Y.

   b. Determine the total utility that will be generated per unit of time for this individual.