CHAPTER 3 LECTURE - CONSUMER PREFERENCES AND THE CONCEPT OF UTILITY

Our objective is to construct a simple model of consumer behavior that will permit us to predict consumers' reactions to changes in their opportunities and constraints. We will take tastes and preferences as given, but we will represent them with a very general analytical model.

Consumer Preferences

Utility: theoretical concept that represents the level of satisfaction or enjoyment that a consumer receives from consumption of a good.

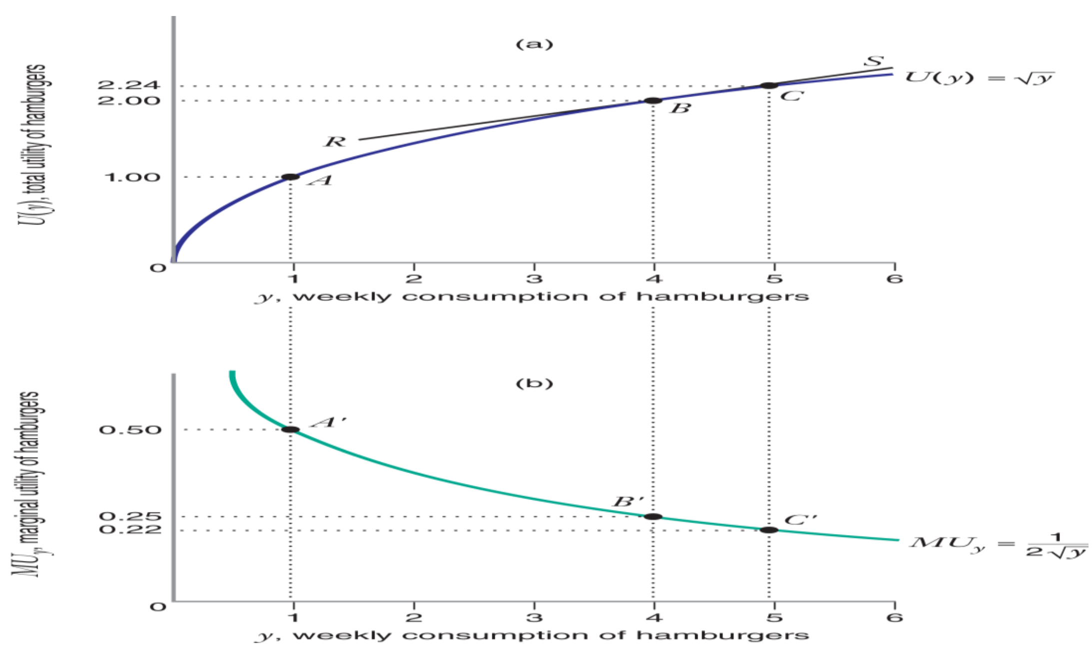
We do not measure utility. Consumers do not measure their utility in any units of measure, but they can rank their utilities from different consumption bundles.

Ordinal Theory – places market basket in the order of most preferred to least preferred, but doesn't indicate by how much one market basket is preferred to another.

Cardinal Utility Theory - quantitatively measuring a consumer's satisfaction, but is mostly a theoretical concept.

We define **Total Utility** as the utility that a consumer receives from all of the units of a particular good that she consumes. Define **Marginal Utility** as the increase in Total Utility that corresponds to a one-unit increase in consumption of a good.

Diminishing Marginal Utility: plays a very important role in our analysis of consumer behavior.



As the quantity of a good consumed increases (ceteris paribus), the marginal utility attached to consuming additional units of the good eventually diminishes.

Looking at the mathematics:

We define Marginal utility (y for example) as simply: $MU_Y = \frac{\Delta U}{\Delta Y} = \frac{dU}{dY} = \frac{\partial U}{\partial Y}$.

Suppose we have a U=f(Y) = $U(Y) = \sqrt{Y} = Y^{\frac{1}{2}}$

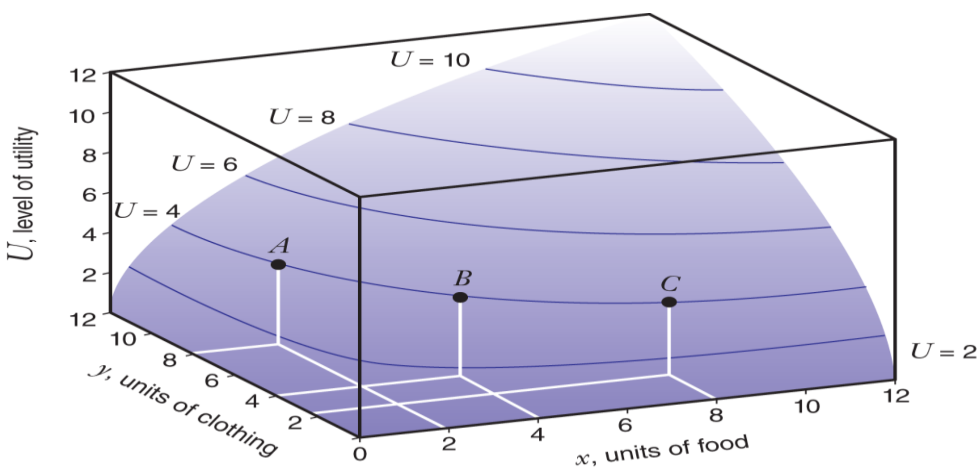
Solving for MUy $MU_Y = \frac{dU}{dY} = \frac{1}{2}Y^{-\frac{1}{2}} = \frac{1}{2\sqrt{Y}}$

Is this subject to diminishing marginal utility? Yes. HOW WOULD YOU PROVE? S.O.C.

$\frac{dMU_Y}{dY} = \frac{d^2U}{dY^2} = -\frac{1}{4}Y^{-\frac{3}{2}} < 0$ S.O.C for maximization satisfied.

Is More Always Better?

PREFERENCES WITH MULTIPLE GOODS: MARGINAL UTILITY, INDIFFERENCE CURVES, AND THE MARGINAL RATE OF SUBSTITUTION



Suppose the Utility Function is $U = X^{\frac{1}{2}}Y^{\frac{1}{2}}$. The level of utility is shown on the vertical axis, and the amounts of food (x) and clothing (y) are shown, respectively, on the right and left axes. Contours representing lines of constant utility are also shown. For example, the consumer is indifferent between baskets A, B, and C because they all yield the same level of utility (U = 4)

We previously defined Marginal utility in the one good case (y for example) as: $MU_Y = \frac{\Delta U}{\Delta Y} = \frac{dU}{dY} = \frac{\partial U}{\partial Y}$.

In the multiple good case, for example where U =U(X,Y)

$MU_X \,|\, Y \text{ held constant} = \frac{\Delta U}{\Delta X} = \frac{dU}{dX} = \frac{\partial U}{\partial X}$ $MU_Y \,|\, X \text{ held constant} = \frac{\Delta U}{\Delta Y} = \frac{dU}{dY} = \frac{\partial U}{\partial Y}$

If $U = X^{\frac{1}{2}}Y^{\frac{1}{2}}$ $\quad MU_X \,|\, Y\ held\ constant = \frac{\partial U}{\partial X} = \frac{1}{2}X^{-\frac{1}{2}}Y^{\frac{1}{2}}$ $\quad MU_Y \,|\, X\ held\ constant = \frac{\partial U}{\partial Y} = \frac{1}{2}X^{\frac{1}{2}}Y^{-\frac{1}{2}}$

*You can show that both X and Y are subject to diminishing marginal utility***.**

INDIFFERENCE CURVES

The First Steps in Understanding Utility – Make some assumptions about people/individuals.

Premises of the model:

1. Individual ***tastes*** or ***preferences*** determine the amount of pleasure people derive from the goods and services they consume.
2. Consumers face ***constraints***, or limits, on their choices.
3. Consumers ***maximize*** their well-being or pleasure from consumption subject to the budget and other constraints they face.

Assumption One: **Completeness**.

We assume that an individual has *preferences over any two bundles of goods* (a bundle can be a single good or a bunch of goods). In other words, they can choose between them or decide they are indifferent. Prefer A to B or Prefer B to A or be Indifferent

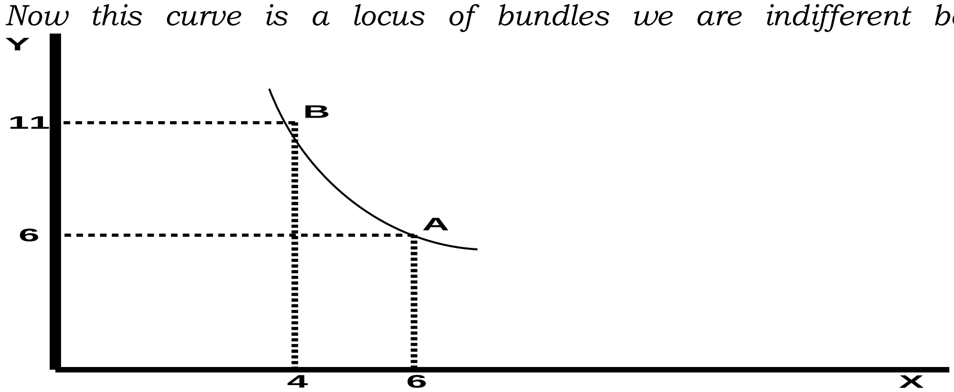
Assumption Two: **More is better**.

This is simply that if a person considers something to be a good (i.e. they value it) then more of it is preferred to less.

From these two assumptions alone, we can construct an indifference curve.

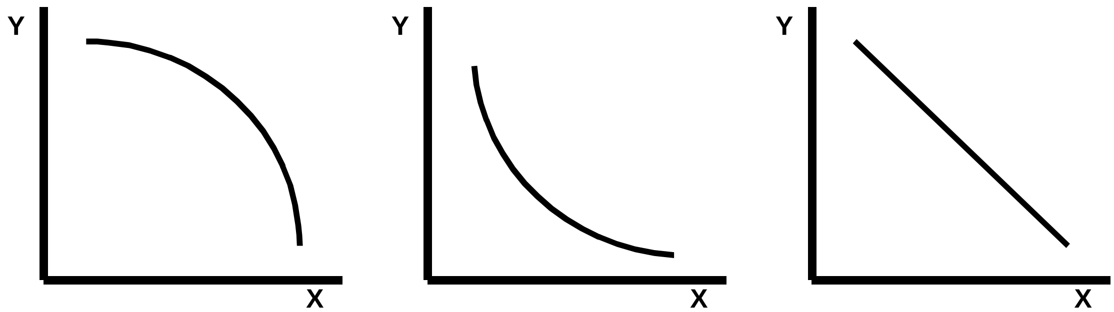
An **Indifference Curve** is a line (curve) that shows all the possible combinations of two goods between which a person is indifferent. In other words, it shows the consumption of different combinations of two goods that will give the same utility (satisfaction) to the person.

Now this curve is a locus of bundles we are indifferent between. Let's graph this.



So what do we know? Well, that tells us that an indifference curve must be ***negatively sloped***; in other words, must go from the Northwest to the southeast. But it doesn't tell us much about the shape of the curve. **Is it concave (bowed out), convex (bowed in), or straight?**

What do these curves show?



For that we need our third postulate.

Assumption Three: **Diminishing Marginal Rate of Substitution**

What that means in English is that as I acquire ***more and more*** of a good I am willing to give up ***less and less*** of other goods to obtain it. That implies an indifference curve will be convex (or bowed in) – the middle graph above.

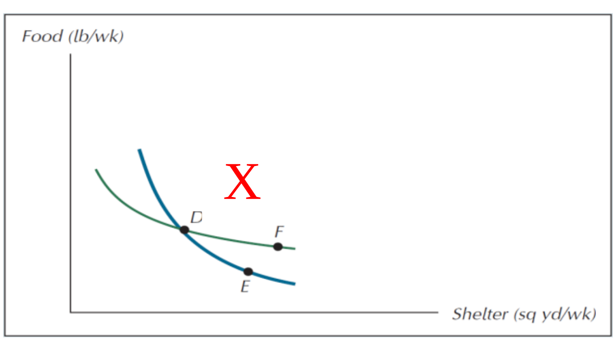
We'll will come back to that to discuss why that is the case.

There is one more thing we need to know about indifference curves and that comes from the fourth assumption.

Assumption Four: **Transitivity**

What does that mean? Well, it means my preferences or choices are consistent. In other words, if I prefer A to B and B to C, then I also prefer A to C. Or equivalently, I am indifferent between A and B…and indifferent between B and C, then I am also indifferent between A and C.

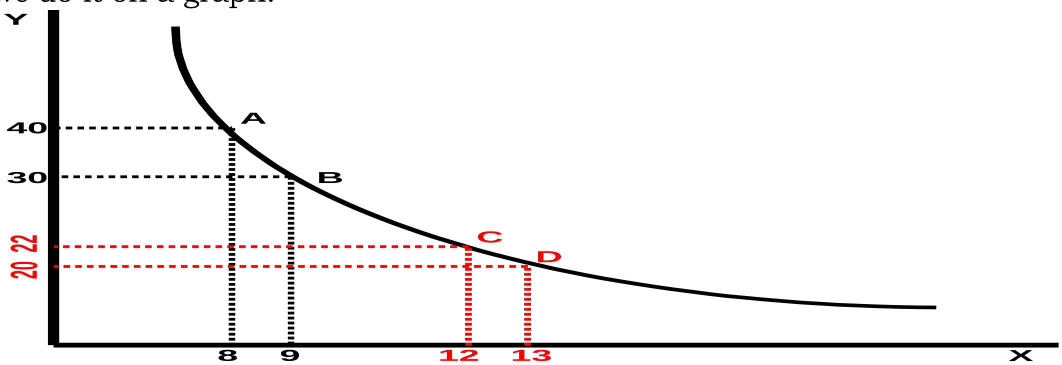
What this implies is that indifference curves can never touch or cross each other.



Interpreting the Indifference Curves

Interpreting/reading indifference curves takes us back to that third assumption of diminishing marginal rate of substitution. ***What is the marginal rate of substitution?***

This is the size of the reduction in the variable on the vertical axis that leaves the individual indifferent following an increase of one unit of the variable on the horizontal axis. This is much clearer if we do it on a graph.



Another way of saying it is that **the MRS is the maximum amount a person will give up to obtain one more unit.** The maximum amount is the amount that leaves him just indifferent.

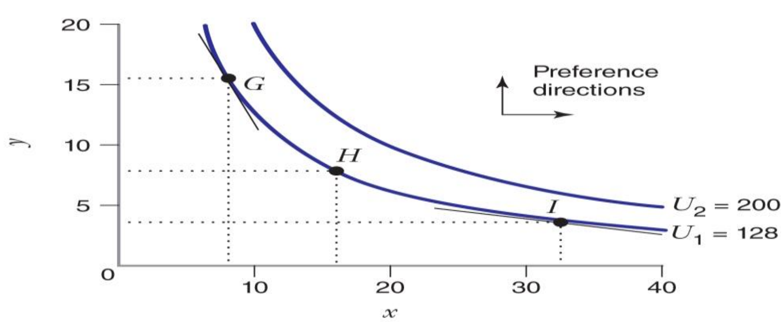
This should ring a bell in regard to demand: remember how we interpreted the height of a demand curve. It was the maximum amount a person would pay – in effect, how much he valued that unit. Well, we'll link all this up.

So this is what we mean by Marginal Rate of Substitution – *but what about this diminishing part?*

The diminishing part comes from what happens as this individual consumes more and more of X. As you can see from the slope, it is getting flatter. *What this means is that as he has more X, the maximum amount of Y he is willing to give up to get more X gets less and less.*

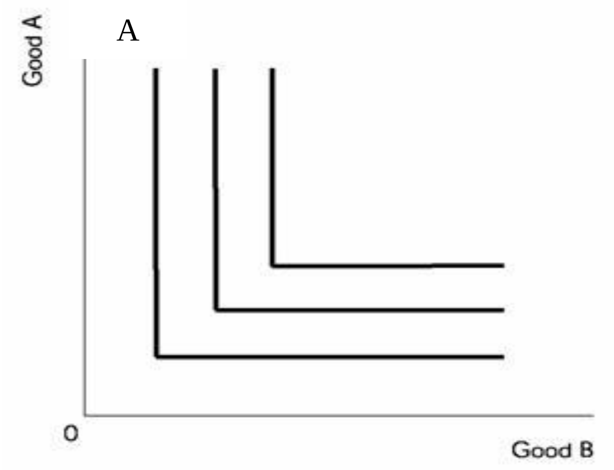
Up at the top (steep slope), he is willing to give up 10Y (go from 40Y to 30Y) to get one more X (from bundle A to bundle B) but down here, he is only willing to give up 2Y to get one more X (from bundle **C** to bundle D). The rate I am willing to substitute in X for Y is diminishing as I consume more X.

A good example of an indifference map

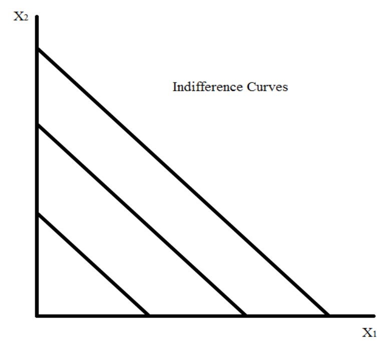


Other Examples - How would you interpret indifference curves that looked like this?

A



B



Look at A. In this case you must consume these two goods in fixed ratios – like one left and one right shoe, or one set of eye glass frames and one set of lenses – things like that.

When looking at the graph you can see that having one right shoe and two left shoes that the additional left shoe doesn't make you any better off.

Look at B - These are perfect substitutes

The Slope of the Indifference Curve

Along an indifference curve, we say that **utility is constant** -- in other words, if I am indifferent between two bundles that means it gives me the same level of utility.

Therefore, we have the following: $MU_X\Delta X + MU_Y\Delta Y = 0$

The first terms is the addition in utility resulting from additional X (so $\Delta X>0$); the second term is the decrease in utility resulting from the decrease in Y (so $\Delta Y<0$).

Rearranging $MU_X\Delta X = -MU_Y\Delta Y$.

Remember, marginal rate of substitution is the slope of the indifference curve, which is $\Delta Y/\Delta X$ (rise over the run). So as you can see $MRS = \frac{\Delta Y}{\Delta X} = -\frac{MU_X}{MU_Y}$. **So the marginal rate of substitution is simply the ratio of the marginal utilities.**

$U = U(X, Y)$ *Taking total differential*

$$dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy = 0 \qquad MU_X dx + MU_Y dy = 0 \qquad \frac{dy}{dx} = -\frac{MU_X}{MU_Y}$$

Utility and the Marginal Rate of Substitution

Suppose $U = X^{\frac{1}{2}}Y^{\frac{1}{2}}$ We can see that $MU_X = \frac{\partial U}{\partial X} = \frac{1}{2}X^{-\frac{1}{2}}Y^{\frac{1}{2}}$ and $MU_Y = \frac{\partial U}{\partial Y} = \frac{1}{2}X^{\frac{1}{2}}Y^{-\frac{1}{2}}$

$MRS = \frac{MU_X}{MU_Y} = \frac{\frac{1}{2}X^{-\frac{1}{2}}Y^{\frac{1}{2}}}{\frac{1}{2}X^{\frac{1}{2}}Y^{-\frac{1}{2}}} = \frac{Y}{X}$ You can see that as we give up Y for X, MRS falls.

If the utility function is

$U(x,y) = x + xy + y$ what is the MRS? Is the indifference curve convex?

$MU_X = \frac{\partial U}{\partial X} = 1 + y$ and $MU_Y = \frac{\partial U}{\partial Y} = x + 1$ $MRS = \frac{MU_X}{MU_Y} = \frac{1+y}{x+1}$

You can see the MU's are constant.

If the utility function is $U(x,y) = U = X^2Y^2$ what is the MRS? Is the indifference curve convex?

Are the marginal utilities of X and Y subject to decreasing, increasing or constant MU?

More Examples of Utility Functions

1) Cobb Douglas Utility Function

In general form it may be written as $U = X^{\alpha}Y^{\beta}$ or $U = X^{\alpha}Y^{1-\alpha}$ (special case)

Using calculus to find the MU's (general form of Cobb-Douglas function).

$U = X^{\alpha}Y^{\beta}$

$MU_X = \frac{\partial U}{\partial X} = \alpha X^{\alpha-1}Y^{\beta}$ $MU_Y = \frac{\partial U}{\partial Y} = \beta X^{\alpha}Y^{\beta-1}$

From above we can calculate the MRS.

MRS = $\frac{MP_X}{MP_Y} = \frac{\alpha X^{\alpha-1}Y^{\beta}}{\beta X^{\alpha}Y^{\beta-1}} = \frac{\alpha Y}{\beta X}$. This is constant.

The Cobb-Douglas function can also be written as $\ln U = \alpha \ln X + \beta \ln Y$

Can you answer this problem?

Suppose a consumer's preferences for two goods can be represented by the Cobb–Douglas utility function $U = AX^{\alpha}Y^{\beta}$, where A, α, and β are positive constants. You can show the marginal utilities are

$$MU_X = \frac{\partial U}{\partial X} = \alpha AX^{\alpha-1}Y^{\beta} \quad \text{and} \quad MU_Y = \frac{\partial U}{\partial Y} = \beta AX^{\alpha}Y^{\beta-1}$$

Answer the following for this utility function.

a) Is the assumption that more is better satisfied for both goods? **Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.**

b) Does the marginal utility of x diminish, remain constant, or increase as the consumer buys more x? Explain.

$$\frac{\partial^2 U}{\partial X^2} = (\alpha - 1)\alpha AX^{\alpha-2}Y^{\beta}$$

Since we do not know the value of α, only that it is positive, we need to specify three possible cases:

1. **When $\alpha < 1$, the marginal utility of x diminishes as x increases.**
2. **When $\alpha = 1$, the marginal utility of x remains constant as x increases.**
3. **When $\alpha > 1$, the marginal utility of x increases as x increases.**

c) What is the $MRS_{x,y}$? $MRS_{X,Y} = \frac{\alpha AX^{\alpha-1}Y^{\beta}}{\beta AX^{\alpha}Y^{\beta-1}} = \frac{\alpha Y}{\beta X}$

d) Is $MRS_{x,y}$ diminishing, constant, or increasing as the consumer substitutes x for y along an indifference curve? **As the consumer substitutes x for y, the $MRS_{x,y}$ will diminish.**

2) Linear Utility Function

In general form it may be written as U = aX + bY

The linear nature of this utility function implies that the goods are perfect substitutes.

Using calculus to find the MU's of X and Y

$MU_X = \frac{\partial U}{\partial X} = a \qquad MU_Y = \frac{\partial U}{\partial Y} = b \qquad$ We can see the $MRS_{x,y} = \frac{a}{b}$

Can you answer this problem?

Consider the utility function $U(x, y) = 3x + y$, with $MU_x = 3$ and $MU_y = 1$.

a) Is the assumption that more is better satisfied for both goods? **Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.**

b) Does the marginal utility of x diminish, remain constant, or increase as the consumer buys more x? Explain. **The marginal utility of x remains constant at 3 for all values of x.**

c) What is $MRS_{x, y}$? $\boldsymbol{MRS_{x,y} = 3}$

d) Is $MRS_{x, y}$ diminishing, constant, or increasing as the consumer substitutes x for y along an indifference curve? **The $\boldsymbol{MRS_{x,y}}$ remains constant moving along the indifference curve.**

Can you answer this problem?

Consider the utility function $U = x^2 + y^2$. The marginal utilities are $MU_x = 2x$ and $MU_y = 2y$.

a) Is the assumption that more is better satisfied for both goods? **Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.**

b) Does the marginal utility of x diminish, remain constant, or increase as the consumer buys more x? Explain. **The marginal utility of x increases as the consumer buys more x.**

c) What is $MRS_{x, y}$? $MRS_{X,Y} = \frac{2x}{2y} = \frac{x}{y}$

d) Is $MRS_{x, y}$ diminishing, constant, or increasing as the consumer substitutes x for y along an indifference curve? **As the consumer substitutes x for y, the $\boldsymbol{MRS_{x,y}}$ will increase.**

3) Constant Elasticity of Substitution (CES) Utility Function.

$$U = \left[X^{\rho} + Y^{\rho} \right]^{1/\rho}$$

The value ρ is a number between -∞ and 1. This is called a constant elasticity of substitution (CES) utility function. CES functions are also used in production theory. You should be able to show that the marginal utilities for this utility function are given by

$$MU_x = [X^{\rho} + Y^{\rho}]^{1/\rho - 1} X^{\rho-1} \qquad MU_y = [X^{\rho} + Y^{\rho}]^{1/\rho - 1} Y^{\rho-1}$$

Does this utility function exhibit the property of diminishing $MRS_{x,y}$?

Recall that $MRS_{x,y} = \frac{MU_x}{MU_y}$. Substituting in the marginal utilities given above yields

$$MRS_{X,Y} = \frac{x^{\rho-1}}{y^{\rho-1}}$$

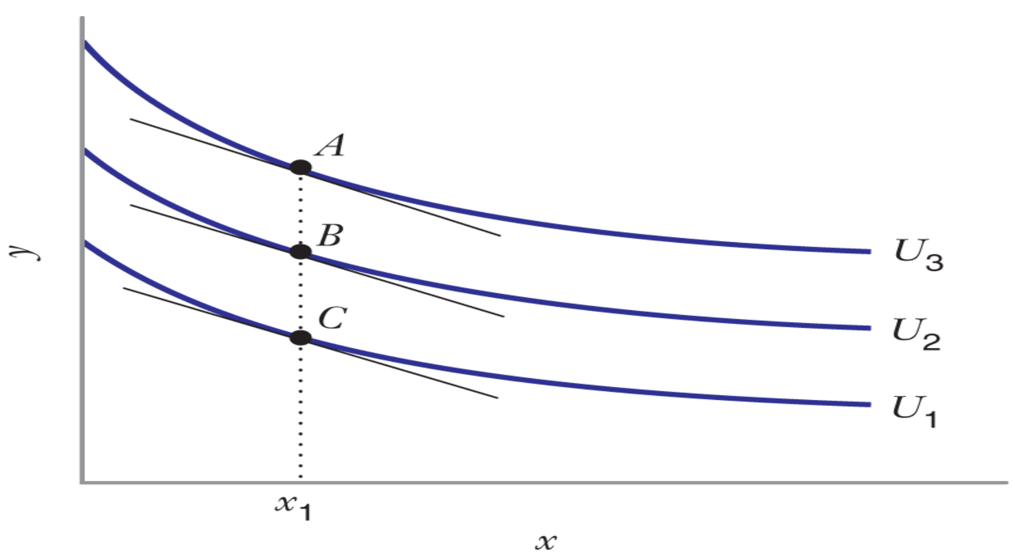
Now, because $\rho < 1$, $x^{\rho - 1}$ decreases as x increases. By the same logic, $y^{\rho - 1}$ increases as y decreases. As we "slide down" an indifference curve, x increases and y decreases, so it follows that $MRS_{x,y}$ decreases. Thus, this utility function exhibits diminishing marginal rate of substitution of x for y.

- For $\rho = 1$: $U(x,y)$ corresponds to perfect substitutes case
- For ρ approaching 0: $U(x,y)$ approaches similar to Cobb-Douglas
- For ρ approaching $-\infty$: $U(x,y)$ approaches the case of perfect complements

4) Quasilinear utility function - A utility function that is linear in at least one of the goods consumed, but may be a nonlinear function of the other good(s)

The equation for a quasilinear utility function is U(x, y) = v(x) + by, where b is a positive constant and v(x) is a function that increases in x—the value of v(x) increases as x increases [e.g., v(x) = X^2 or v(x)= $\sqrt{X}$]. This utility function is linear in y, but generally not linear in x. That is why it is called quasilinear.

Below are indifference curves for a quasilinear utility function. The distinguishing characteristic of a quasilinear utility function is that, as we move due north on the indifference map, the marginal rate of substitution of x for y remains the same. That is, at any value of x, the slopes of all of the indifference curves will be the same, so the indifference curves are parallel to each other.



Additional Notes

1) Cobb-Douglas (CD) General Form:

$U = X^{\alpha}Y^{\beta}$ or $U = X^{\alpha}Y^{1-\alpha}$ (special case), where α and β are constants.

Shortcut for finding the MRS: For Cobb-Douglass utility functions, it is always true that

MRS = $\frac{\alpha Y}{\beta X}$. The indifference curves are always strictly convex (that is, the MRS diminishes).

2) Perfect complements General form:

U = min{ax,by}, where a and b are constants.

Examples: U = min{x,y}
U = min{2x,y}
U = min{2x,4y}
U = min{(1/3)x,(1/4)y}

Finding the MRS: The utility function is not differentiable, so we cannot use calculus techniques to find the MRS. However we know that the ICs are "L"-shaped, so all we need to know in order to draw them is the point at which they are kinked. It is always the case that these kinked points of the IC lie on a line whose equation you can derive by setting ax = by.

3) Perfect substitutes General form:

U = ax + by, where a and b are constants.

Examples: U = x + y
U = 2x + y
U = 2x + 3y
U = (1/2)x + (3/4)y

Finding the MRS: The MRS is always equal to a/b, a constant. That is, ICs are straight lines with slope (negative) a/b.

4) Quasi-linear General form:

U = ax + f(y) or U = f(x) + ay, where a is a constant.

Examples: U = 2x + ln y
U = 0.5x + $y^{0.5}$
U = 6 ln x + y
U = $2x^{0.5}$ + (3/4)y

Finding the MRS: No shortcut for finding the MRS: you will need to take the ratio of partial derivatives in every case.