Practice and Study Questions for Chapters 10 -14
(with partial solutions)

1. If each competitive firm in an industry has the short-run cost function \( TC = 50 + 5q + q^2 \), and the market price is $35 (We will do in Class)
   a. What is the profit-maximizing output level for each firm?
   b. What is the total revenue and what are the profits?

2. Suppose, in Question 1, that fixed costs were $250 instead of $50. How does this change affect the firm’s output decision and profits? Should the firm continue to operate? (We will do in Class)

3. In a competitive industry consisting of 10,000 firms, the short-run marginal cost curve for each firm is given by \( MC = 200 + 30Q \). The demand curve faced by the industry is given as 
   \( P = 400 - .002Q \).
   Find the equilibrium price and quantities for the industry and each firm.

Solution

The supply curve for the industry is equal to the sum of the individual marginal cost curves:

\[
\text{We need to sum the individual } MC \text{ – but you have to make sure you sum the Q's}
\]

Individual \( MC = 200 + 30Q_i \)

Solving for individual \( Q_i \)
\[
Q_i = -200/30 + MC/30
\]

Q = 10,000 x \( Q_i = -2,000,000/30 + 10,0000/30MC \)

Solve now for MC which will be same as \( P \)

\[
(10,0000/30)MC = 2,000,000/30 + Q
\]

Divide all by \( 30/10,000 \)

\[
MC = 200 + (30/10,000)Q \text{ or } MC = 200 + .003Q
\]

Set demand equal to supply: \( 400 - .002Q = 200 + .003Q \)
200 = .005Q: so \( Q = 200/.005 = 40,000 \) for the industry
Q for firm = 40,000/10,000 = 4
\( P = 200 + 30(4) = 320 \)
4. If the long run total costs for each firm in a competitive industry are given by:

\[ \text{LTC}(Q) = 2(Q)^3 - 12(Q)^2 + 25Q, \]
with long run marginal costs given as:
\[ \text{LMC} = 6(Q)^2 - 24Q + 25, \]
what is the long run equilibrium price for the industry?

\[ \text{Solution:} \]
Set \( \text{AC} = \text{MC} \)
\[ \text{AC} = \frac{\text{TC}}{Q} \quad \text{AC} = \frac{(2Q^3 - 12Q^2 + 25Q)}{Q} = 2Q^2 - 12Q + 25 \]
\[ 2Q^2 - 12Q + 25 = 6Q^2 - 24Q + 25 \quad 4Q^2 = 12Q \quad Q = 3 \]
Price = marginal cost \( \text{MC} = 6(3)^2 - 24(3) + 25 = 54 - 72 + 25 = 7 \)

5. Find the elasticity of supply in Problem 4.

\[ \text{Solution:} \]
\[ \text{Elasticity of supply} = \frac{(P/Q)(1/slope)}{(320/40,000)(1/.003)} = 2.67 \]

6. What are the conditions for profit maximization for a monopoly? Compare these conditions with those for perfect competition. How do these conditions differ? Look at second order conditions.

7. Understand the effects of taxation on both the perfectly competitive firm and the monopolist. What type of tax affects output? What type of tax does not affect output? **WE DID IN CLASS. Look at Lecture Notes**

8. Understand the concept of game theory as discussed in class. Be able to define a dominant strategy and Nash equilibrium. Understand the prisoner’s dilemma. What is a maximin strategy? **Look at Notes**

9. What do we mean by a natural monopoly? What factors can account for the existence of a natural monopoly. What would be the socially optimum level of output in this model? What is a compromise solution? **Look at Notes**

10. The market demand curve for a pair of Cournot duopolists is given as:
\[ P = 36 - 3Q, \quad (Q = Q_1 + Q_2). \]
The constant per unit marginal cost is 18 for each duopolist. Find the Cournot equilibrium price, quantity, and profits.

\[ \text{Solution:} \]
\[ P = 36 - 3Q, \quad Q = Q_1 + Q_2, \quad \text{MC} = 18 \]
\[ \text{TR}_1 = 36(Q_1 + Q_2) - 3Q_1^2 - 3Q_2Q_1 \]
MR₁ = 36 - 6Q₁ - 3Q₂
Set MC = MR and solve for Q₁
18 = 36 - 6Q₁ - 3Q₂
6Q₁ = 18 - 3Q₂
Q₁ = 3 - 1/2Q₂

We can now solve for Q₂

You can show Q₂ = 3 - 1/2Q₁
Q₁ = 3 - 1.5 + 0.25Q₁
0.75Q₁ = 1.5 or Q₁ = 2
Q = 4

P = 36 - 3(4) = 36 - 12 = 24
Profit = PQ - MC(Q) = 24(2) - 18(2) = 48 - 36 = 12 for each firm.

11. Solve Problem 10 as a Bertrand model. Find the long-run equilibrium price, quantities, and profits.

Solution:
P = MC = 18
18 = 36 - 3Q
Q = 18/3 = 6
Q₁ = Q₂ = 6/2 = 3
Profit = PQ - MC(Q) = 18(6) - 18(6) = 0
Profit (firm 1) = profit (firm 2) = 0

12. Solve Problem 10 as a Stackelberg Leader-Follower model. Assume firm 1 is the leader.

Solution:
Stackelberg model:
Follower (firm 2):
MR = 36 - 3Q₁ - 6Q₂ = 18
Q₂ = 3 - ½ Q₁

Leader (firm 1):
P₁ = 36 - 3Q₂ - 3Q₁ = 36 - 3(6-Q₁/2) -3Q₁
P₁ = 36 - 18/2 + 3Q₁/2 - 3Q₁ = 27 - 3/2Q₁
MR₁ = 27 - 3Q₁
Set MR₁ = MC
27 - 3Q₁ = 18
3Q₁ = 9
Q₁ = 3

Q₂ = (6 - Q₁)/2 = (6 - 3)/2 = 3/2
Q = Q₁ + Q₂ = 3 + 3/2 = 9/2
P = 36 - 3(9/2) = 36 - 13.5 = 225
Profit (firm 1) = P Q₁ - MC(Q₁) = 22.5(31) - 18(3) = 67.5 - 54 = 13.5
Profit (firm 2) = P₂ - MC(Q₂) = 22.5(1.5) - 18(1.5) = 6.75
13. You are the manager of a firm producing coffee. The marginal product of labor is $MP_L = 72L^{-1/2}$. Suppose that the firm is a competitor in the coffee market. The price of green coffee is $1 per pound. Further suppose that the firm is a competitor in the labor market. The wage rate is $12.00 per hour.

a. Given the information above what is the marginal revenue product of labor?

b. How much labor should the firm hire?

**We did in class** \( L = 36 \)

14. Suppose a firm has an internal wage structure that can be described by the equation $Wage/hr. = 8 + .2(VMP_L)$. Only the workers with a VMP/hr. of _____ will be paid exactly equal to their marginal product (assume Price level is equal to 1). What will these workers be paid? \( W = 10 \)