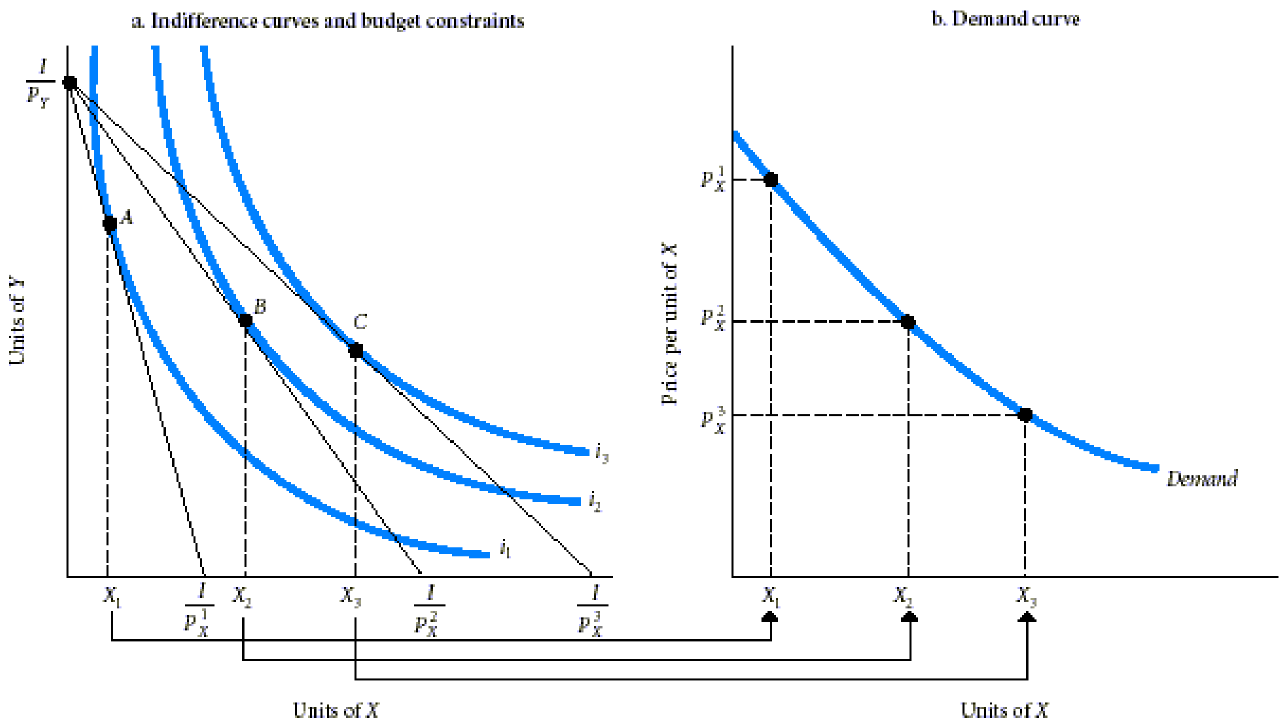
CHAPTER 4 LECTURE - INDIVIDUAL AND MARKET DEMAND

Deriving a Demand Curve - Using the graph below we can derive a demand curve.

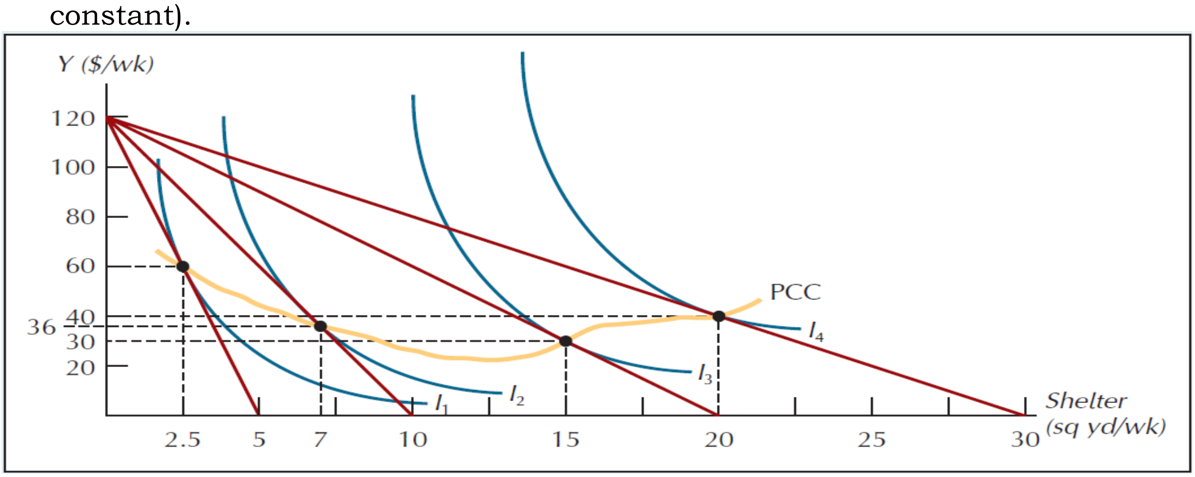
Remember along a demand curve, what is constant? Income, preferences, price of other goods, etc. -- the only things that change are price and quantity.

The following graph depicts that situation where the price of X has fallen and total expenditure has risen.

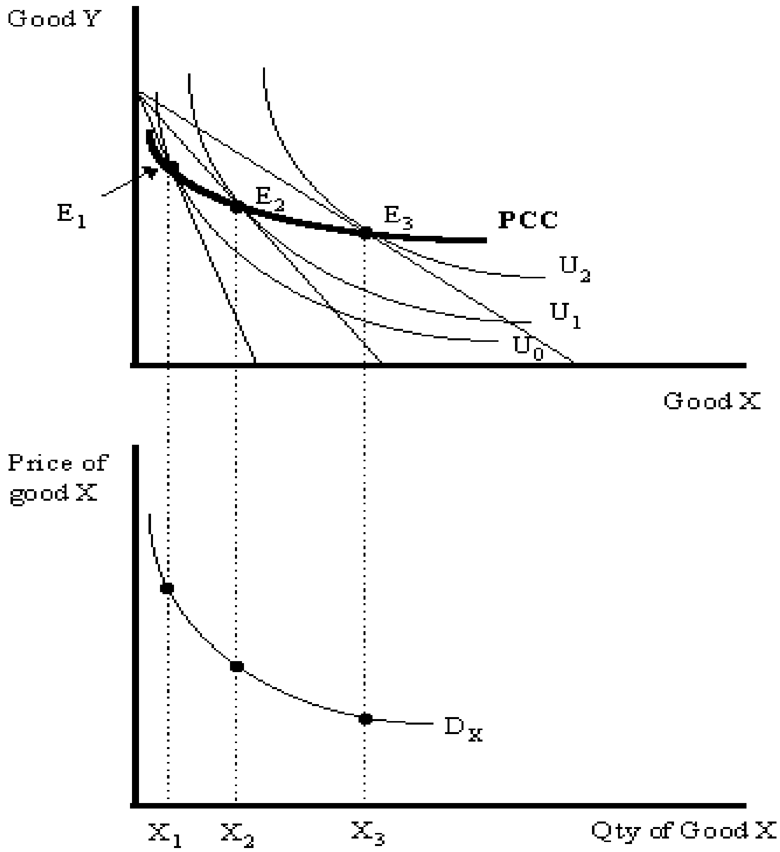


The Price Consumption Curve

Price-Consumption Curve (PCC): for a good X is the set of optimal bundles traced on an indifference map as the price of X varies (holding income and the price of Y constant).



Derivation of Demand Curve Again



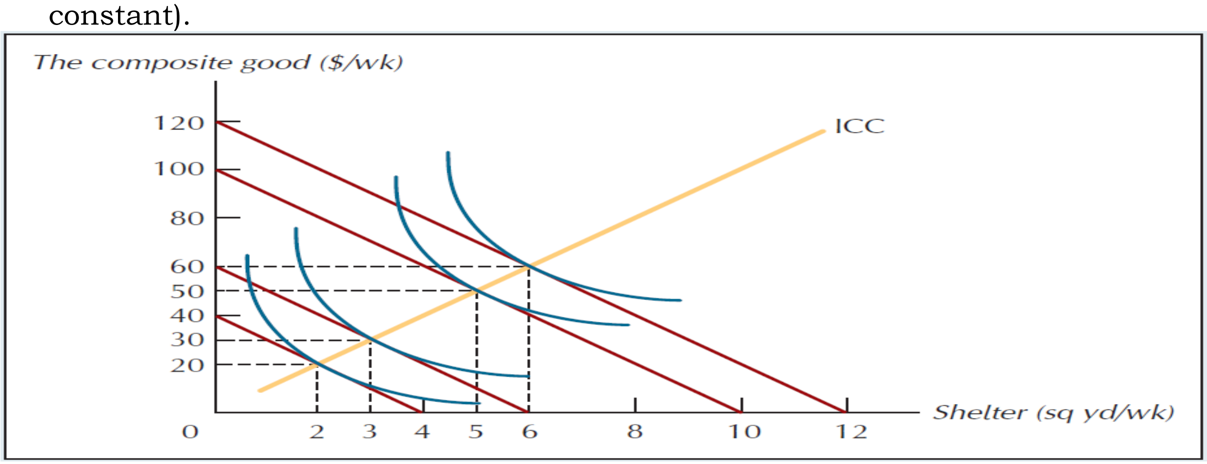
A change in the price of one of the goods, *ceteris paribus*, will clearly change the position (and slope) of the budget line leading to a new equilibrium bundle. Hence the amount consumed of each good will change. The following diagram shows how to derive the individual's demand curve for good X.

How to derive a demand curve
As PX falls the optimum point changes. The line drawn through the optimum points is called a price consumption curve (PCC)

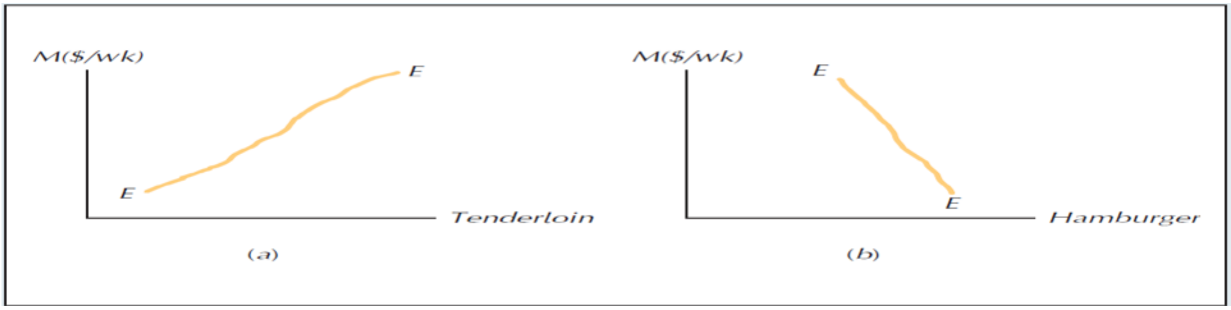
As PX falls the quantity the individual would like to consume of good X rises. Hence, in the bottom half of the diagram, we can derive the individual's demand curve for good X given income, price of good Y and preferences. The model of rational consumer choice predicts that an individual's demand curve for a good is downward-sloping

Effect of Changes in Income: Income Consumption Curve
(Also called 'offer curve')

Income-Consumption Curve (ICC): for a good X is the set of optimal bundles traced on an indifference map as income varies (holding the prices of X and Y constant).



***Engel Ccurve**:* curve that plots the relationship between the quantity of X consumed and income.



→ Engel curve showing demand against income

Normal goods: demand increases with income
Inferior goods: demand falls with income

Income and Substitution Effects

Substitution Effect: that component of the total effect of a price change that results from the associated change in the relative attractiveness of other goods.

Income Effect: that component of the total effect of a price change that results from the associated change in real purchasing power.

Total Effect: the sum of the substitution and income effects.

Total Effect of a Price Change is the change in quantity demanded as the consumer moves from one equilibrium consumption bundle to another.

Total Effect = Substitution Effect + Income Effect

= ΔQ_d caused by a Δ relative price, holding real income stable. + ΔQ_d caused by a Δ real income, holding relative price stable

Substitution Effect - The change in quantity demanded given a change in the relative price after compensating for the change in real income (which is caused by the change in relative prices).

1. In general, the substitution effect is negative.
   a. If relative price falls for a good, then quantity demanded increases.
   b. If relative price increases for a good, then quantity demanded falls.

2. Graphically it is shown by keeping the consumer on the same indifference curve, but with the new relative price ratio!

Income Effect - The change in quantity demanded of a good resulting solely from a change in real income (which is caused by the change in relative prices).

1. In general, the income effect is positive.
   a. If real income increases, consumers buy more of the good.
   b. If real income falls, consumers buy less of the good.

2. Normal and superior goods have a positive income effect.

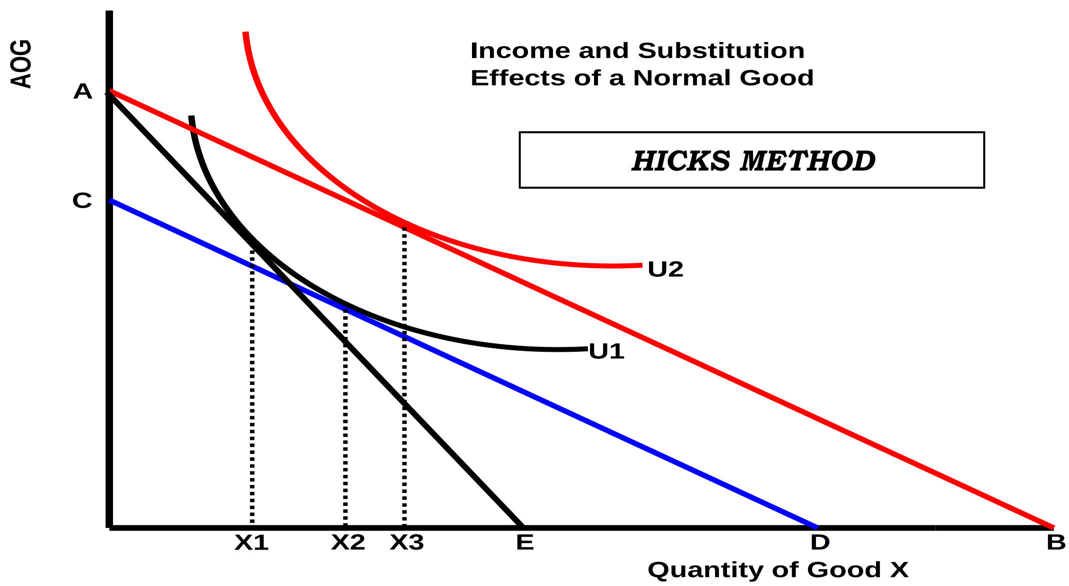
3. Inferior goods have a negative income effect.

4. Graphically it is shown by moving the consumer to a higher or lower indifference curve.

In general, a positive income effect reinforces a negative substitution effect.

We will do this by graphically breaking down a price decrease.

Consider the following graph:



AE is the original budget constraint; a price decrease of X rotates the budget line out to AB. The consumer increases her consumption of X from X1 to X3.

Now comes the tricky part. Two things have happened with the price decrease; X has become cheaper relative to other goods (this is the substitution effect); plus, this person can buy more of ***all goods***, including X (this is the income effect).

Consider the following mental experiment.

Let's take away just enough income so that the person is on the same indifference curve as she was ***before*** the price decrease. This we will define as the **income effect** of a price change. Now find the equilibrium point of consumption -- on the graph this is depicted as X2.

Note, even though she is on the same original indifference curve she is still consuming more X. The difference between X1 and X2 is what we call the **substitution effect**.

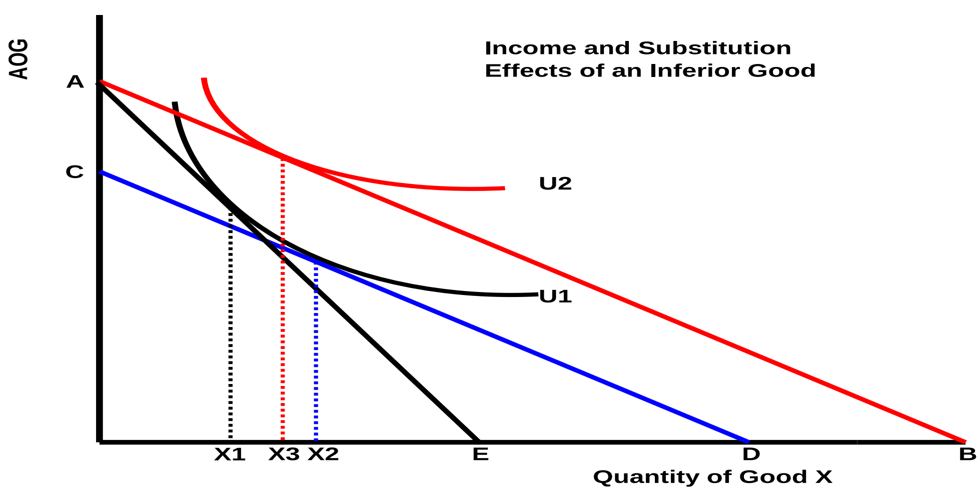
A way to think about the substitution effect is the following: Suppose you are on a strict budget for either you parent, guardians or partner and part of your consumption is soda (good X). Suddenly the price of beer drops -- your consumption of soda would increase (to X3) and you are better off (on indifference curve U2). However, you stupidly tell those responsible for giving you money and in response they reduce your budget (lower your income). And after they reduce your budget you find you are just as well off as you were before (back on U1) but you are consuming soda than from before the price change (X2).

So now we have broken down the price change into its component parts. The Total Effect has her increasing her consumption from X1 to X3. The Substitution Effect is from X1 to X2 and the Income Effect is from X2 to X3.

Note: the two effects are in the same direction -- when price decreases, the substitution effect is positive (buys more); the income effect is also positive -- ***but does it have to be?***

No, it doesn't. It is in this case because **we have assumed the good is normal**.

Let's depict what would happen if the good were inferior; this we show in the following graph:



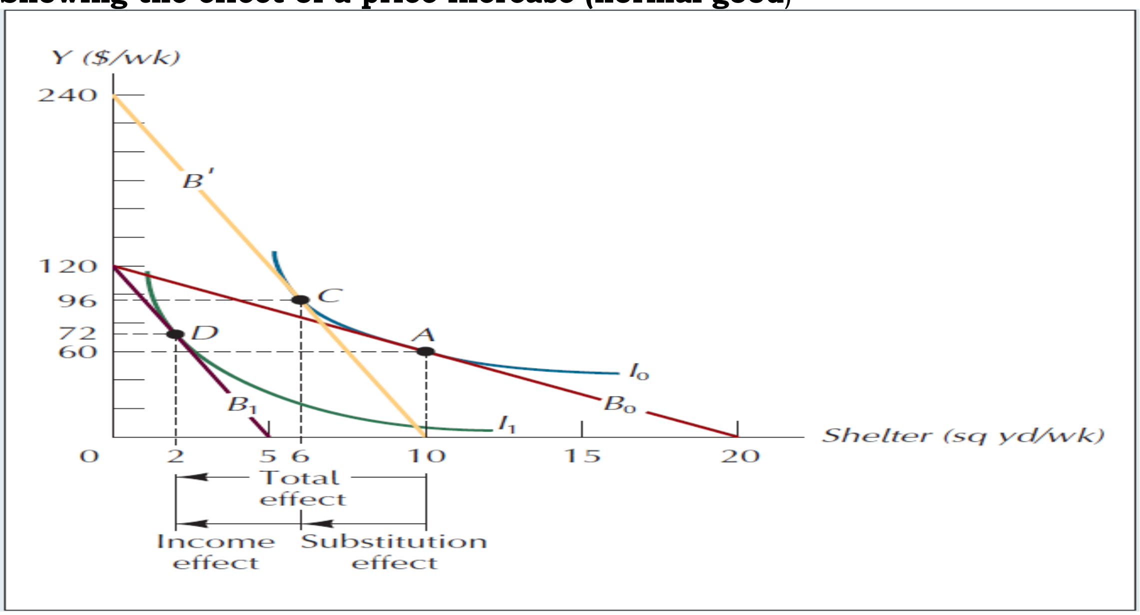
Everything is the same as last time except we're going to assume the good is inferior. This doesn't affect the substitution effect; it is exactly the same as before, from X1 to X2. But as we increase the income (in effect, shifting the budget constraint out from CD to AB), we decrease our consumption of X (from X2 to X3).

Do we still have a downward sloping demand curve?

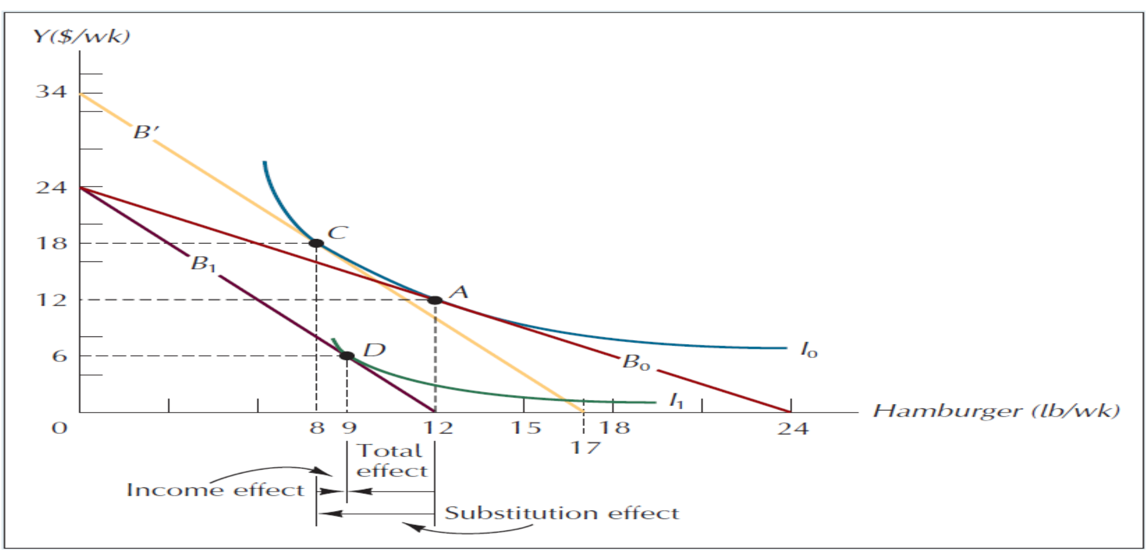
Yes, we do. The total effect, from X1 to X3 is still positive showing that our consumption of X increases as the price decreases.

The reason it is positive is that the substitution effect is greater than the income effect.

Showing the effect of a price increase (normal good)



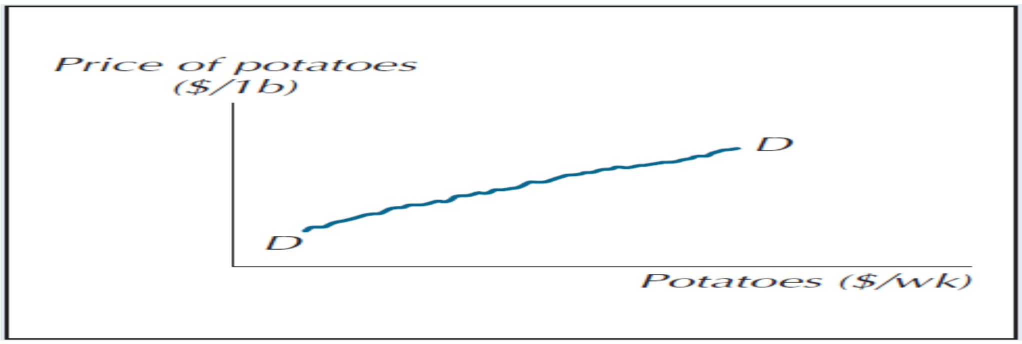
Showing the effect of a price increase (inferior good)



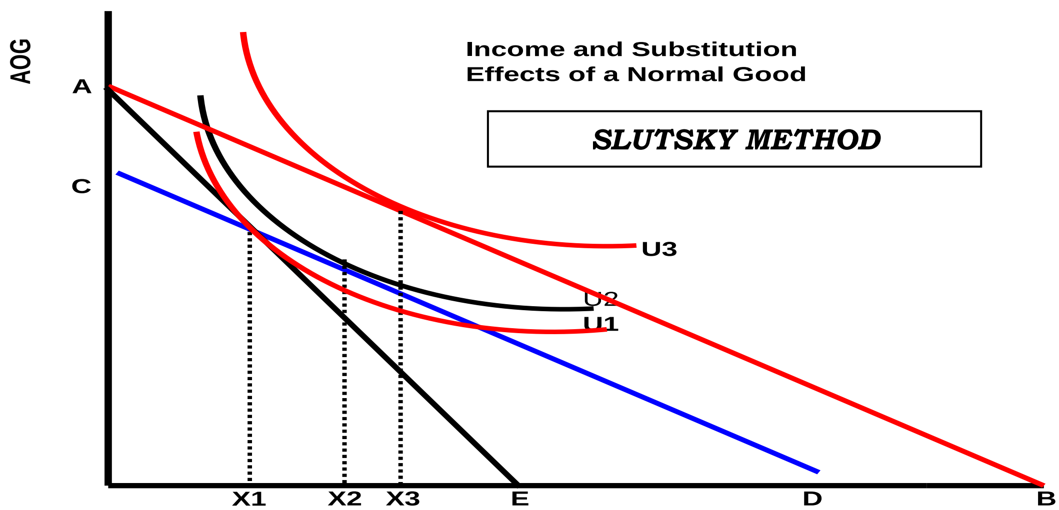
Conditions for an Upward Sloping Demand Curve –Giffen Good

First, it must be an inferior good. Second, for that inferior good the income effect must be greater than the substitution effect. For the latter condition to hold, the price change must be for a good that consumes a large part of your total budget. (For example: my scotch expenses take up a large part of my overall budget, so the price of scotch went down it is possible my consumption of scotch could go down -- if scotch was an inferior good, fortunately scotch is normal good to me.)

Not only does a good have to be inferior -- which is relatively rare -- but also the good has to take up a 'large' portion of a person's total budget -- which is even rarer. This is why we do not observe upward sloping demand curves even though they are theoretically possible.



The above method of showing the income and substitution effect is based on the Hicks Method. That is, utility is held constant. NOTE: SOME TEXTS LOOK AT THE SLUTSKY SUBSTITUTION EFFECT. IN THIS CASE, WE HOLD REAL INCOME CONSTANT. THIS IS SHOWN IN THE GRAPH BELOW.



In this case we take away enough nominal income so real income has not changed. This is at point X1. Here the substitution effect is X1 to X2 and income effect is X2 to X3.

Mathematical Analysis

Looking at deriving demand functions

Maximize: $U = U(X,Y)$ Subject to: $P_y X + P_y Y = M$

Example: Suppose U= XY

Find utility maximizing choice with prices, P_x and P_y income M

Short cut: $P_x/P_y = MU_x/MU_y = Y/X$ (since $MU_x = Y$ and $MU_Y = X$)

so $P_x X = P_y Y$

Looking at the Budget Constraint $M = 2P_x X$ or $M = 2P_Y Y$

Demand curve for x is $X = M/2P_x$ and Demand curve for $Y = M/2Py$

Duality in Consumer Theory

We can look at behavior in two ways:

1. Maximize utility given budget constraint
2. Minimize expenditure necessary to achieve a given level of Utility (U*)

Result is the same no matter how we state problem.

Let E denote expenditure, so $E = P_x X + P_y Y$ Utility function is $U = U(X, Y)$

Minimize expenditure subject to constraint that $U^* = U(X, Y)$

This is a constrained minimization problem, hence, the Lagrangean function is:

$$L = P_X X + P_Y Y + \lambda(U^* - U(X,Y))$$

Taking the derivative of *L* with respect to each good (i.e., *X* and *Y*) and setting it equal to zero:

$$\frac{\partial L}{\partial X} = P_X - \lambda \frac{\partial U}{\partial X} = 0 \qquad \frac{\partial L}{\partial Y} = P_Y - \lambda \frac{\partial U}{\partial Y} = 0$$

Solving for λ: $\lambda = \dfrac{MU_X}{P_X}$ and $\lambda = \dfrac{MU_Y}{P_Y}$

Equating these two equations yields the same result as the before: $\dfrac{MU_X}{P_X} = \dfrac{MU_Y}{P_y}$

Looking once again at deriving demand functions

Expenditure Function: $E = P_x X + P_y Y$

Subject to Constraint: $U(X,Y) = XY = U^*$

Find Expenditure minimizing choice with prices, P_x and P_y , income M and U=XY

Short cut: $P_x/P_y = MU_x/MU_y = Y/X$ (since $MU_x = Y$ and $MU_Y = X$)

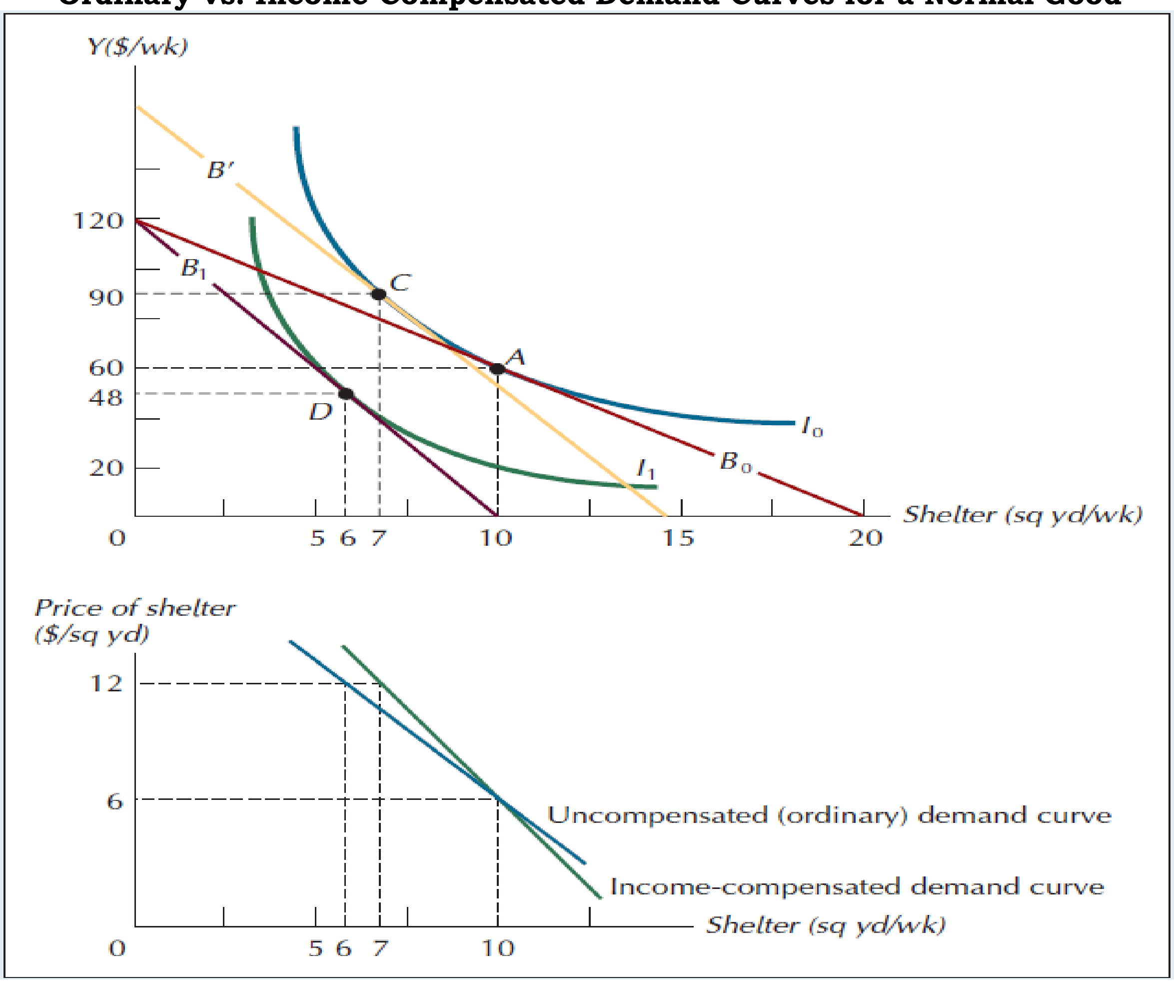
so $P_x X = P_y Y$

Looking at the Budget Constraint E (or M) = $2P_xX$ or M = $2P_YY$

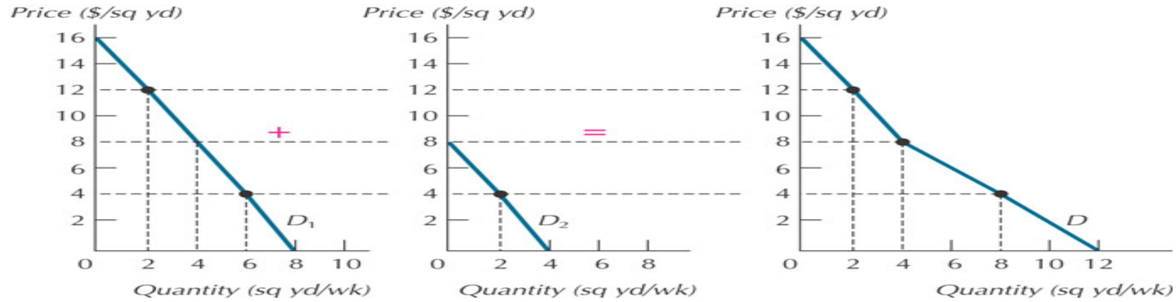
Demand curve for X is $X = M/2P_x$ and Demand curve for $Y = M/2Py$

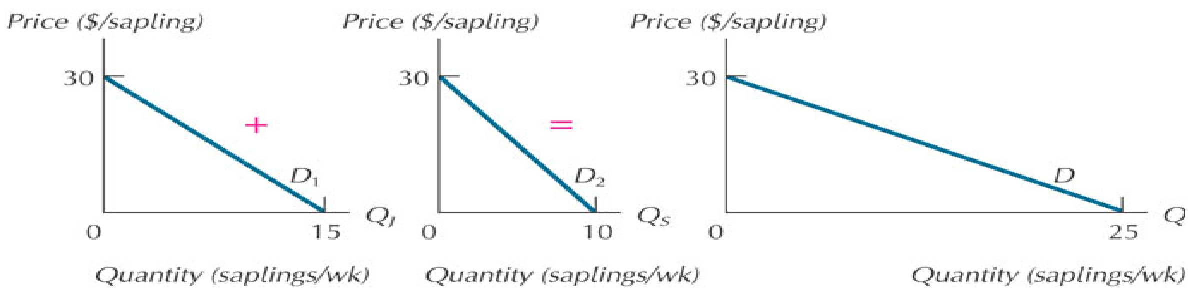
Same result as before.

Ordinary vs. Income-Compensated Demand Curves for a Normal Good



Aggregating Individual Demand Curves





ELASTICITY OF SUPPLY AND DEMAND

ELASTICITY can be defined as a measure of responsiveness. We want to examine how a change in one variable affects a change in another.

Price elasticity of demand is defined as the percentage change in quantity demanded with respect to a percentage change in the price of the good.

The symbol often used to denote price elasticity of demand is E_d even though the text uses € (epsilon). E_d is easier to write so we can us E_d. Other texts may different symbols – Economists are not consistent). We will use the following formula for price elasticity of demand: We keep in mind that Q= Q_d

$E_d = \frac{\%\Delta Q}{\%\Delta P} = \frac{\Delta Q}{\Delta P} x \frac{P}{Q}$ or if we use calculus we can define elasticity as

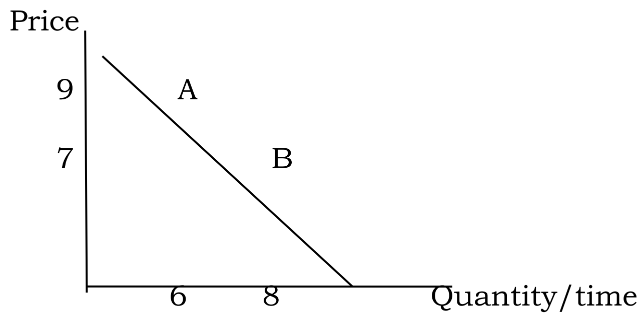
$E_d = \frac{dQ}{dP} x \frac{P}{Q} = \frac{\partial Q}{\partial P}\frac{P}{Q}$. We will look at using this formula a little later.

Some books put a negative sign in front of the equation or take the absolute value, the value of elasticity always becomes positive. Some books do not consider the absolute value and treat elasticity as negative. **BE AWARE OF HOW ELASTICITY IS DEFINED**.

Notice that the value of the price elasticity of demand includes the reciprocal of the slope of the demand function, $\Delta P/\Delta Q_d$. The value of the slope of the demand function is a factor affecting the value of the elasticity. However, the values are not the same.

Calculating Price Elasticity of Demand by the Arc Elasticity Method

Consider the demand curve below.



Calculate the value of the elasticity between points A and B.

$\Delta Q = 8 - 6 = 2$ $\Delta P = 7 - 9 = -2$

We will always look at the absolute value of elasticity.

If we use price and quantity at point A, then $E_d = \frac{\Delta Q}{\Delta P} x \frac{P}{Q} = \frac{2}{-2} x \frac{9}{6} = -1.5$

If we use price and quantity at point B, then $E_d = \frac{\Delta Q}{\Delta P} x \frac{P}{Q} = \frac{2}{-2} x \frac{7}{8} = -0.87$

It can be seen that as we move down along the demand curve the value of the elasticity changes. Thus, the value of the elasticity we calculate from A to B will be different depending on which initial values of P and Q are used. To cope with this, we take the average of price and quantity and use the formula:

$E_d = \frac{\Delta Q}{\Delta P} x \frac{P^*}{Q^*}$ where $P^* = \frac{P_1 + P_2}{2}$ and $Q^* = \frac{Q_1 + Q_2}{2}$

Using a little bit of algebra, the formula above reduces to:

$E_d = \frac{\Delta Q}{\Delta P} x \frac{P_1 + P_2}{Q_1 + Q_2}$ Using the data from before: $E_d = \frac{\Delta Q}{\Delta P} x \frac{P_1 + P_2}{Q_1 + Q_2} = \frac{2}{-2} x \frac{9+7}{6+8} = -1.14$

If we have a straight line demand curve we can use determine elasticity at a particular point.

Let Q = 10 – 2P What is the value of the elasticity at P= 2?

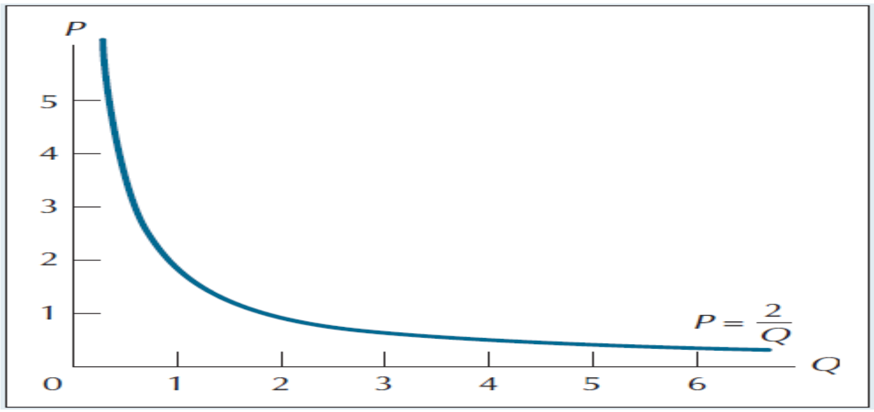
Solving for $\frac{dQ}{dP}$ we obtain $\frac{dQ}{dP}$ = -2, and at P= 2, Q = 10 – 2 (2) = 6.

Thus, $\frac{dQ}{dP} x \frac{P}{Q}$ = $(-2) x \frac{2}{6} = -0.67$

Special Case of a constant price elasticity demand curve

Let $Q = aP^{-b}$ or $Q = \frac{a}{P^b}$ $\quad$ a and b are constants

$\frac{dQ}{dP} = -baP^{-b-1}$ $\quad$ If P = P, $Q = \frac{a}{P^b}$ $\quad$ Thus, $E_d = (-baP^{-b-1})\frac{P}{aP^{-b}} = -b$



Elasticity of Demand is constant at any given price.

If the demand function is in logarithmic form then the coefficient of the variable is the value of the elasticity.

Let Q = $2P^{-3}$ $\quad$ Taking the natural logs of both sides yields ln Q = ln2 – 3ln P

We know that the derivative of a logarithm is

If y = lnx

$\frac{dy}{dx} = \frac{d\ln x}{dx} = \frac{1}{x}$ $\quad$ or $\quad d\ln x = \frac{dx}{x}$ $\quad$ so $\quad d\ln Q = \frac{dQ}{Q}$ $\quad$ and $\quad d\ln P = \frac{dP}{P}$

Elasticity is defined as $E_d = \frac{\frac{dQ}{Q}}{\frac{dP}{P}} = \frac{d\ln Q}{d\ln P}$

From before: ln Q = ln2 – 3ln P $\quad E_d = \frac{d\ln Q}{d\ln P} = (\frac{d\ln 2}{d\ln P} - 3\frac{d\ln P}{d\ln P} + \ln P\frac{d\ln -3}{d\ln P}) = (-3) = -3$

Knowing how to calculate the value of the price elasticity of demand is important. What is perhaps more important, however, is understanding what the value of elasticity means and its relationship to the concept of **TOTAL REVENUE**.

TOTAL REVENUE is defined as price times quantity.

We noted before that the value of elasticity indicates the percentage change in quantity with respect to the percentage change in price. Since elasticity is a fraction, the value calculated can be interpreted as the percentage change in quantity with respect to a one percent change in price.

If, for example, E_d = -1.14, a one percent increase in price would cause quantity to fall by 1.14 percent. This can also be interpreted as saying a ten percent increase in price would cause quantity to fall by 11.4 percent.

Elasticity and Total Revenue (TR)

When discussing the price elasticity of demand it is useful to distinguish different ranges of elasticity values.

If $|E_d| > 1$, demand is referred to a being elastic.

If $|E_d| < 1$, demand is referred to a being inelastic.

If $|E_d| = 1$, demand is referred to a being unit or unitary elastic.

Whether demand is elastic, inelastic or unit elastic will determine how a change in price (and the resultant change in quantity) will affect total revenue (PQ).

If $|E_d| > 1$, or demand is elastic, a decrease in price will increase the value of total revenue. An increase in price will decrease the value of total revenue.

If $|E_d| < 1$, or demand is inelastic, a decrease in price will decrease the value of total revenue. An increase in price will increase the value of total revenue.

If $|E_d| = 1$, demand is unit elastic, and any change in price will have no effect on total revenue. Thus, total revenue is constant. At the price and quantity in which demand is unit elastic, total revenue is also maximized.

Elasticity and Marginal Revenue (MR)

From above, it was shown that the value of elasticity determines the effect of a price (quantity) change on total revenue. A change in total revenue with respect to a change in quantity is called **Marginal Revenue (MR)**.

Marginal Revenue = $\frac{\Delta TR}{\Delta Q_d} = \frac{dTR}{dQ_d}$ (We will let $Q = Q_d$ so = $MR = \frac{\Delta TR}{\Delta Q} = \frac{dTR}{dQ}$

Deriving the relationship between MR and TR.

TR = PQ Look at changes: $\Delta TR = P\Delta Q + Q\Delta P$

$$MR = \frac{\Delta TR}{\Delta Q} = P\frac{\Delta Q}{\Delta Q} + Q\frac{\Delta P}{\Delta Q} = P + Q\frac{\Delta P}{\Delta Q}$$

Look at the term $\frac{Q\Delta P}{\Delta Q}$. Multiplying by $\frac{P}{P}$

yields $\frac{PQ\Delta P}{P\Delta Q}$

Remembering that $E_d = \frac{\Delta Q}{\Delta P}\frac{P}{Q}$ $MR = P + P(\frac{1}{E_d})$ $MR = P(1+\frac{1}{E_d})$

This can also be solved be taking the derivative of TR with respect to Q.

$$\frac{dTR}{dQ} = P\frac{dQ}{dQ} + Q\frac{dP}{dQ} \quad = \quad MR = P + Q\frac{dP}{dQ} = P + \frac{Q}{P}\frac{dP}{dQ}P$$

$$MR = P(1+\frac{1}{E_d}) \quad \text{or} \quad MR = P(1+\frac{1}{E_d})$$

NOW REMEMBER THAT E_d will be negative. However, it is generally easier to look at the absolute values of E_d or $|E_d|$

Substituting into the MR function yields:

If $|E_d| > 1$, $\{(1+\frac{1}{E_d})\}$ > 0 or $(1-\frac{1}{|E_d|})$ >0., and MR > 0. As P rises (Q falls), TR falls.

If $|E_d| < 1$, $\{(1+\frac{1}{E_d})\}$ < 0, or $(1-\frac{1}{|E_d|})$ <0 and MR < 0. As P rises (Q falls), TR rises.

If $|E_d| = 1$ $E_d = 1$, $\{(1+\frac{1}{E_d})\}$ = 0, $(1-\frac{1}{|E_d|})$ =0 and MR = 0. As P rises (Q falls), TR remains constant.

We can also look a the relationships of changes in Price, Price Elasticity and Total Revenue

$$TR = Q_x \bullet P_x.$$

Taking the derivative of the above total revenue equation with respect to price (dTR/dP_x), we obtain the following general functional relation (you can show):

$$dTR/dP_x = Q_x (1 + E_d)$$

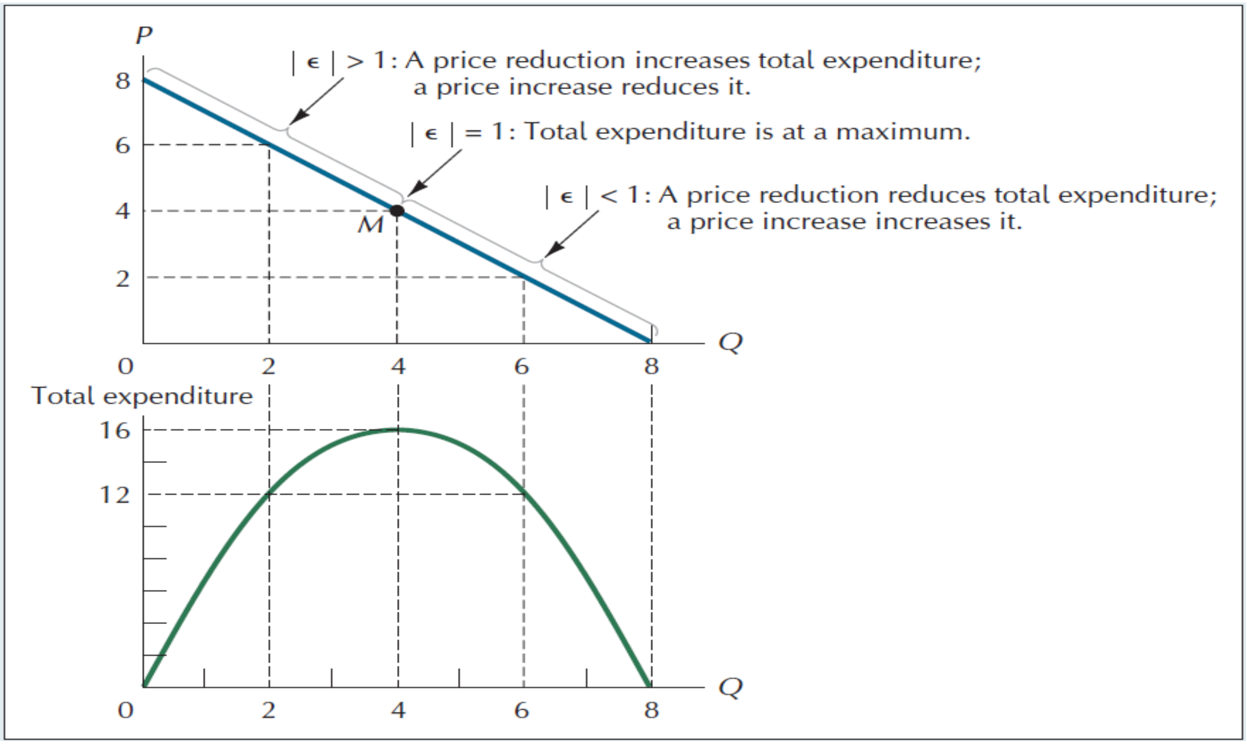
E_d represents the price elasticity of demand. Since E_d is always negative,

$$\mathbf{dTR/dP_x = Q_x (1 - |E_d|)}$$

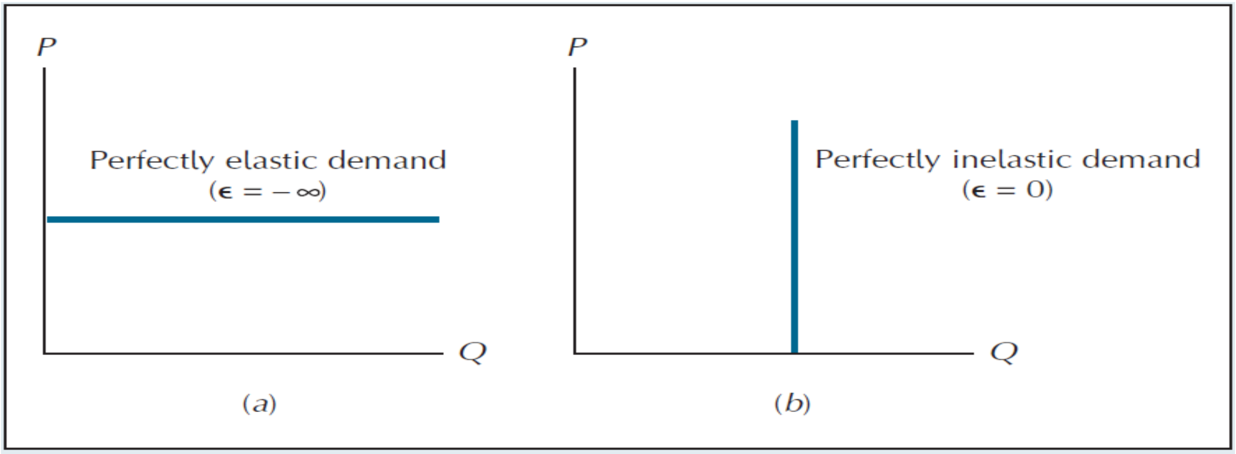
a) If $|E_d| > 1$, then $dTR/dP_x < 0$. In plain English, this says that when demand is price elastic, the relationship between price and total revenue is negative. That is, an increase in price will decrease total revenue and a decrease in price will have the opposite effect on total revenue.

b) If $|E_d| < 1$, then $dTR/dP_x > 0$. Again, in plain English, this says that when demand is price inelastic, the relationship between price and total revenue is positive. That is, an increase in price will have the effect of increasing total revenue and a decrease in price will cause a decline in revenue.

c) If $|E_d| = 1.0$, then $dTR/dP_x = 0$. Thus, a change in price will have no effect on total revenue.



Special Cases of Elasticity



If E_d = - ∞, demand is referred to a being perfectly elastic.

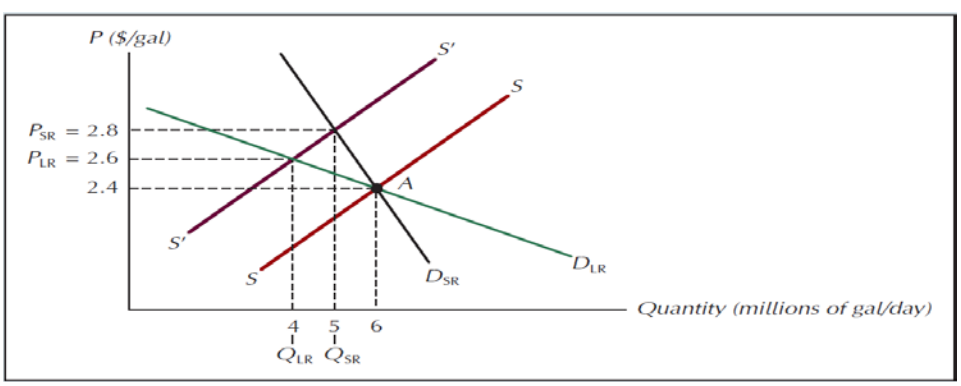
If E_d = 0, demand is referred to a being perfectly inelastic.

NOTE: Although we know that along a (straight line) demand curve the value of elasticity will change, a steep demand curve is often referred to as being relatively inelastic. A flat demand curve is referred to as being relatively elastic.

Factors that affect the price elasticity of demand.

- ***Substitution possibilities:*** the substitution effect of a price change tends to be small for goods with no close substitutes.
- ***Budget share:*** the larger the share of total expenditures accounted for by the product, the more important will be the income effect of a price change.
- ***Direction of income effect:*** a normal good will have a higher price elasticity than an inferior good.
- ***Time:*** demand for a good will be more responsive to price in the long-run than in the short-run.

Price Elasticity Is Greater in the Long Run than in the Short Run



Other Types of Elasticity

1. Cross price elasticity of demand is defined as the percentage change in quantity demanded of good two with respect to a percentage change in the price of good one.

$$E_{1,2} = \frac{\%\Delta Q_2}{\%\Delta P_1} = \frac{\Delta Q_2}{\Delta P_1} x \frac{P_1}{Q_2} \quad \text{or} \quad \frac{dQ_2}{dP_1} x \frac{P_1}{Q_2}$$

If $E_{1,2} > 0$, goods 1 and 2 are substitutes.

If $E_{1,2} < 0$, goods 1 and 2 are complements.

If $E_{1,2} = 0$, goods 1 and 2 are not related.

2. Income elasticity of demand is defined as the percentage change in the quantity of a good with respect to a percentage change in income.

The symbol commonly used to denote income elasticity of demand is μ, the Greek symbol mu. (Although η (eta) is used by the book). The book also uses Y to represent Income, but M or I are also used.

$$\eta = \frac{\%\Delta Q}{\%\Delta I} = \frac{\Delta Q}{\Delta I} x \frac{I}{Q} \quad \text{or} \quad \frac{dQ}{dI} x \frac{I}{Q}$$

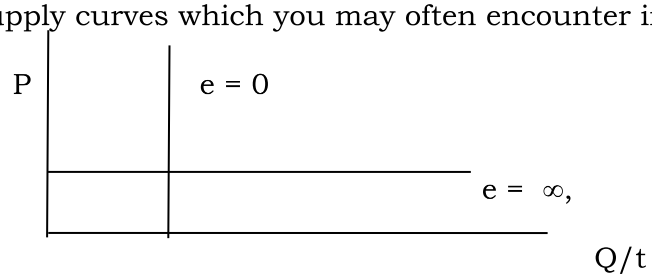
If $0 < \eta \leq 1$, the good is considered a normal good.
If $\eta < 0$, the good is considered an inferior good.
If $\eta > 1$, the good is considered superior

3. Elasticity of Supply is defined as the percentage change in quantity supplied with respect to a percentage change in the price of the good.

The symbol commonly used to denote the price elasticity of supply is the letter e. The formula for elasticity of supply is:

$$e = \frac{\%\Delta Q_s}{\%\Delta P} = \frac{\Delta Q_s}{\Delta P} x \frac{P}{Q_s} \quad \text{or} \quad \frac{dQ_s}{dP} x \frac{P}{Q_s}$$

Supply curves which you may often encounter in economics are:



An example of a good that has a perfectly inelastic supply curve is land. Land is a resource that is in fixed supply and no matter how high price rises output cannot increase.

An example of a resource that has a perfectly elastic supply curve is unskilled labor. Firms can hire all the labor necessary at the going wage rate. In this case, wage is the price of labor and quantity is the amount of labor available.

Range of the Coefficient of Elasticity of Supply: $0 \leq e \leq \infty$

If e > 1 then, elastic price elasticity of supply.
If e = 1 then, unitary elastic price elasticity of supply.
If e > 1 then, inelastic price elasticity of supply.

The elasticity of supply tends to be greater in the long run, when all adjustments to the higher or lower relative price have been made by producers (than in shorter periods of time).

Alternative Market Equilibrium Definitions based on the Price Elasticity of Supply

The definitions are based on the work of Alfred Marshal, who emphasized the time element in competitive price equilibrium.

1. Momentary Equilibrium (**Market Period**)

Producers are totally unresponsive to a price change. Why? There is no time for producers to adjust output levels in response to the change in price!

Supply is perfectly inelastic; therefore, demand determines price.

2. Short-Run Equilibrium

Firms can respond to the change in market price for the good by increasing the variable input in production. That is to say, produces produce more by using their equipment and/or plants more intensively.

Short-run production function:

$Q = F(K, L)$, where K (capital) is the fixed input and L is the variable input.

Hence, firms are able to increase output in the short-run if they increase their variable (labor) input.

3. Long-Run Equilibrium (or "**Normal Price**")

All inputs are variable; hence, firms can increase their capital stock, e.g., build new plants, and new firms can enter the industry or old ones leave. $Q = F(K, L)$.

4. **Very-Long Run Equilibrium**

Technology is improving, so that for a given amount of capital and labor input more output is forthcoming. The supply curve is becoming more elastic! $Q = T\{Q(K, L)\}$.