# INTERMEDIATE MICROECONOMICS LECTURE 3 – RATIONAL CONSUMER CHOICE

Our objective is to construct a simple model of consumer behavior that will permit us to predict consumers' reactions to changes in their opportunities and constraints. We will take tastes and preferences as given, but we will represent them with a very general analytical model.

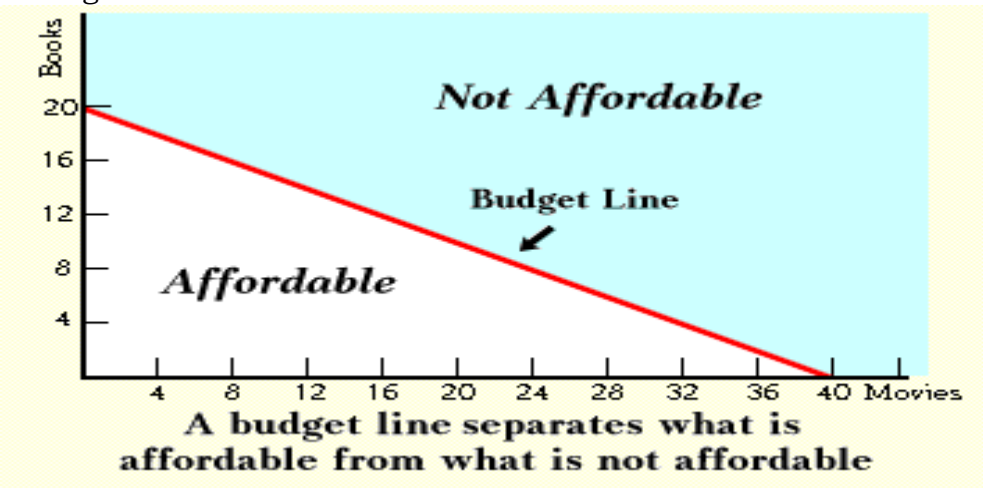
## We will First Look at the Consumer's Opportunity Set or Budget Constraint

General Formulation: Assume for the moment there are only two goods in the world -- this is a simplification we will relax later -- a person therefore will spend all his income on these two goods.

This can be depicted as $M = P_X X + P_Y Y$, where the first term is the person's total expenditure on X and the second term the total expenditure on Y.

Some texts use I for Income.

Let's graph this using some numbers, suppose M = \$200, $P_X$ = \$5, and $P_Y$ = \$10 -- note, as is usually the case, this person is a price taker. From this information we can graph a budget constraint.



A budget line separates what is affordable from what is not affordable

What is the maximum amount of X (Movies) this person can buy? 40 units. How does one know that? From the general formula, (Income or M)/$P_X$ .

a) What is the maximum amount of Y this person can buy? 20 units.
b) Suppose she wants one unit of X -- now what is the maximum amount of Y she could purchase? It would be 18 -- so when you buy ONE X, you give up TWO Y -- this is the REAL or RELATIVE price of X.

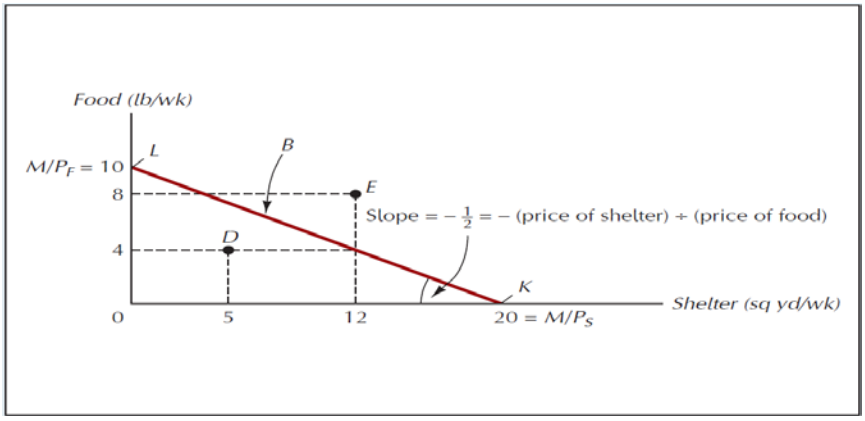
Of course we can put this into a general formula, as follows:

$$M = P_X X + P_Y Y \qquad P_Y Y = M - P_X X \qquad \text{or} \quad Y = \frac{M}{P_Y} - \left(\frac{P_X}{P_Y}\right) X$$

The first term is the intercept of the Y-axis; the second term is the slope. Note **the slope is the price of X over the price of Y -- the relative price or** $-\frac{P_X}{P_Y}$.

We noted that anything within (below and to the left) of the budget line is obtainable; anything beyond the budget line is not obtainable. In this world where we are spending all our income, we will always be on the budget line.

*Look at diagram below. What is the value of M if the price of food is $20.00 and the price of shelter is $10.00?*

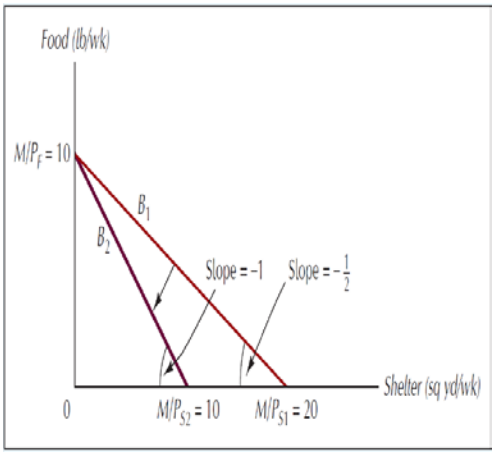



**Changes in the Budget Constraint.**

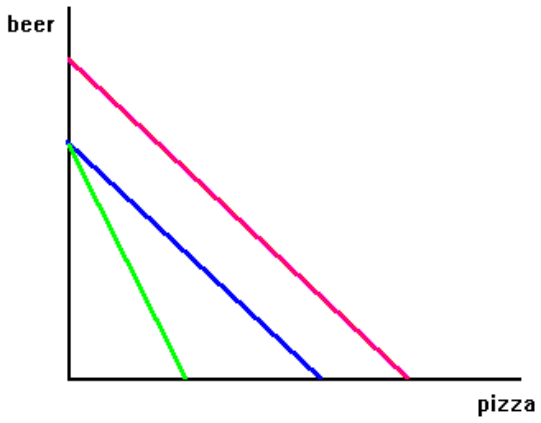
There are three givens when we construct the budget line, income, price of X, and price of Y -- if any of these things change, the budget line changes. What happens if:

**a) Income increases?**
**b) Price of X goes up?**
**c) Price of Y goes down?**
**d) Given a gift certificate of $20 for X?**

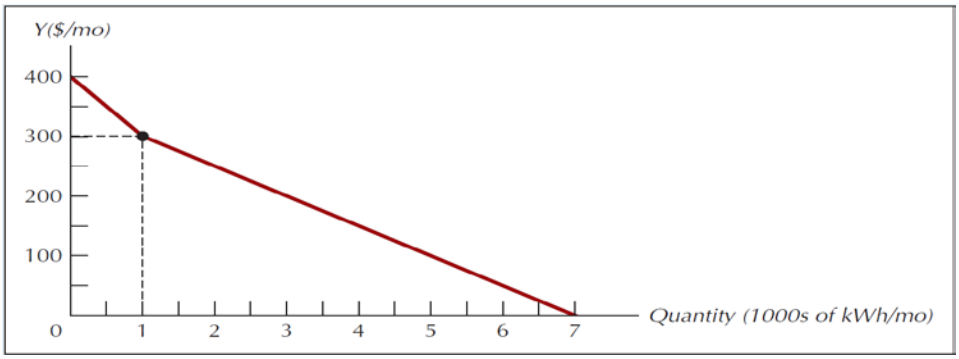
**What has happened here?**




**What has happened here?**



**A Quantity Discount Gives Rise to a Nonlinear Budget Constraint**




## Consumer Preferences

**Utility**: theoretical concept that represents the level of satisfaction or enjoyment that a consumer receives from consumption of a good.

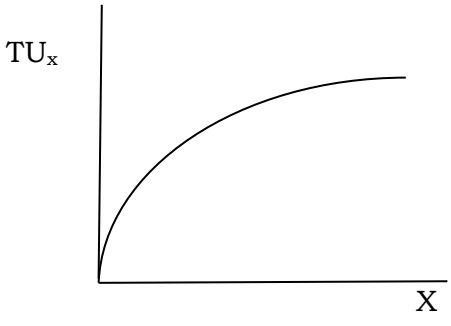
We do not measure utility. Consumers do not measure their utility in any units of measure, but they can rank their utilities from different consumption bundles.

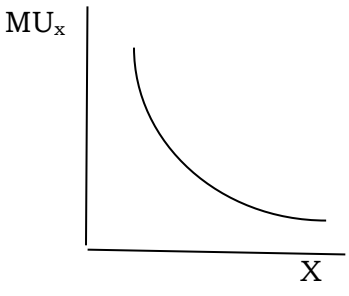
For the moment, define **Total Utility** as the utility that a consumer receives from all of the units of a particular good that she consumes. Define **Marginal Utility** as the increase in Total Utility that corresponds to a one-unit increase in consumption of a good.

**Diminishing Marginal Utility:** plays a very important role in our analysis of consumer behavior.

**As the quantity of a good consumed increases (ceteris paribus), the marginal utility attached to consuming additional units of the good eventually diminishes.**

| $Q_x$ | TU | MU |
|---|---|---|
| 1 | 10 | 10 |
| 2 | 18 | 8 |
| 3 | 24 | 6 |
| 4 | 28 | 4 |
| 5 | 30 | 2 |
| 6 | 30 | 0 |




**Ordinal Theory –** places market basket in the order of most preferred to least preferred, but doesn't indicate by how much one market basket is preferred to another.

**Cardinal Utility Theory -** quantitatively measuring a consumer's satisfaction, but is mostly a theoretical concept.

The First Steps in Understanding Utility – Make some assumptions about people/individuals.

Assumption One: **Completeness**.

We assume that an individual has *preferences over any two bundles of goods* (a bundle can be a single good or a bunch of goods). In other words they can choose between them or decide they are indifferent. Prefer A to B or Prefer B to A or be Indifferent

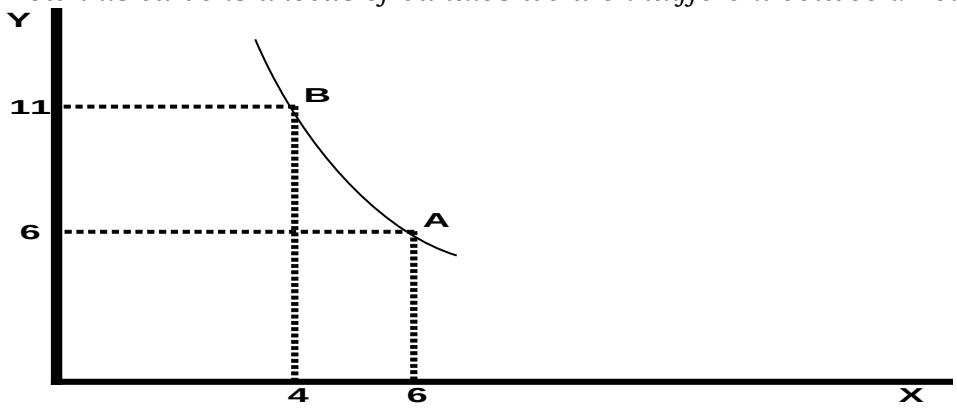
Assumption Two: **More is better**.

This is simply that if a person considers some thing to be a good (i.e. they value it) then more of it is preferred to less.

*From these two assumptions alone we can construct an indifference curve.*

An **Indifference Curve** is a line (curve) that shows all the possible combinations of two goods between which a person is indifferent. In other words, it shows the consumption of different combinations of two goods that will give the same utility (satisfaction) to the person.

*Now this curve is a locus of bundles we are indifferent between. Let's graph this.*




Suppose there are only two goods in the world that we value, X and Y. So in the positive quadrant we can depict all combinations of goods X and Y.

Let's just pick some bundle, A, which consists of 6 units of X and 6 units of Y.

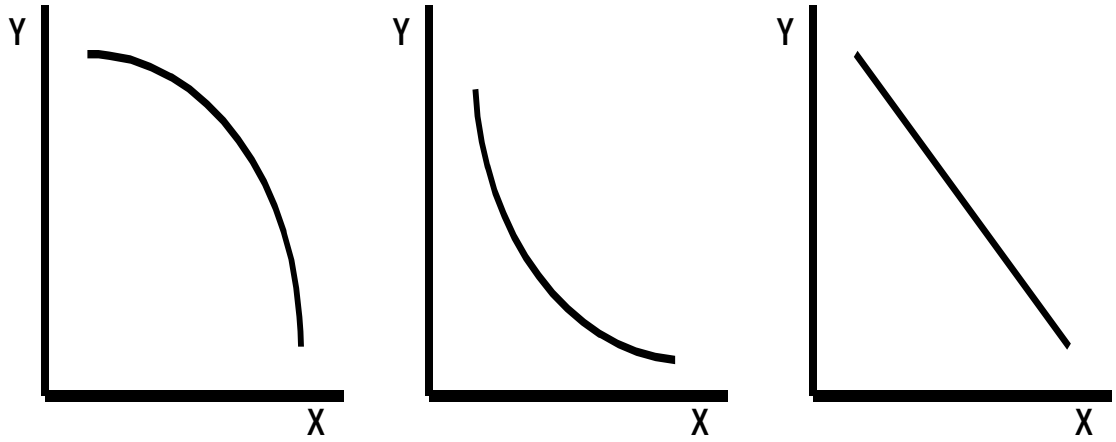
Assumption A tells us that we can compare this bundle with any other bundle no matter where it is – when I say compare I mean we can say, I prefer bundle A to B or B to A or I am indifferent between them.

The second assumption tells us the following, look at Bundle A. Now look at the entire area to the *Northeast* of bundle A….**is there any bundle in that area in which you would prefer bundle A?** NO, because every bundle has at least more of ONE good and never less of one good…therefore, you would always prefer a bundle out there.

Consider the same for any bundles to the southwest – for the same reasons you would always prefer bundle A over ANY bundle in this area.

So what do we know? Well, that tells us that an indifference curve must be ***negatively sloped***; in other words, must go from the Northwest to the southeast. But it doesn't tell us much about the shape of the curve. **Is it concave (bowed out), convex (bowed in), or straight?**

What do these curves show?




For that we need our third postulate.

Assumption Three: **Diminishing Marginal Rate of Substitution**

What that means in English is that as I acquire ***more and more*** of a good I am willing to give up ***less and less*** of other goods to obtain it. That implies an indifference curve will be convex (or bowed in) – the middle graph above.

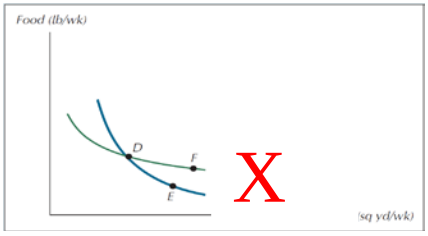
We'll will come back to that to discuss why that is the case.

There is one more thing we need to know about indifference curves and that comes from the fourth assumption.

Assumption Four: **Transitivity**

What does that mean? Well, it means my preferences or choices are consistent. In other words, if I prefer A to B and B to C, then I also prefer A to C. Or equivalently, I am indifferent between A and B…and indifferent between B and C, then I am also indifferent between A and C.

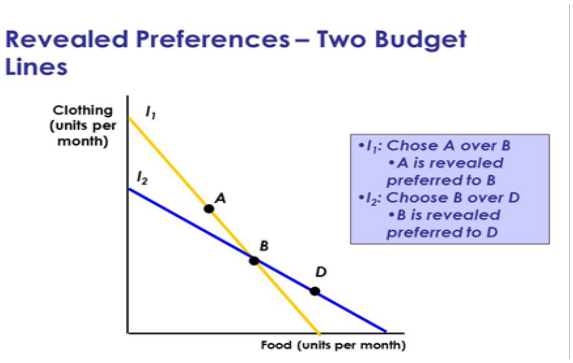
**What this implies is that indifference curves can never touch or cross each other.**



So these postulates allow us to construct indifference curves.

The way we represent a person's preferences is simply a diagram of their indifference curves. This is a very flexible tool; initially we start out with two goods…but of course mathematically we can extend it to as many good as we want.

***Revealed Preference*** - An economic theory of consumption behavior which asserts that the best way to measure consumer preferences is to observe their purchasing behavior. Revealed preference theory works on the assumption that consumers have considered a set of alternatives before making a purchasing decision.

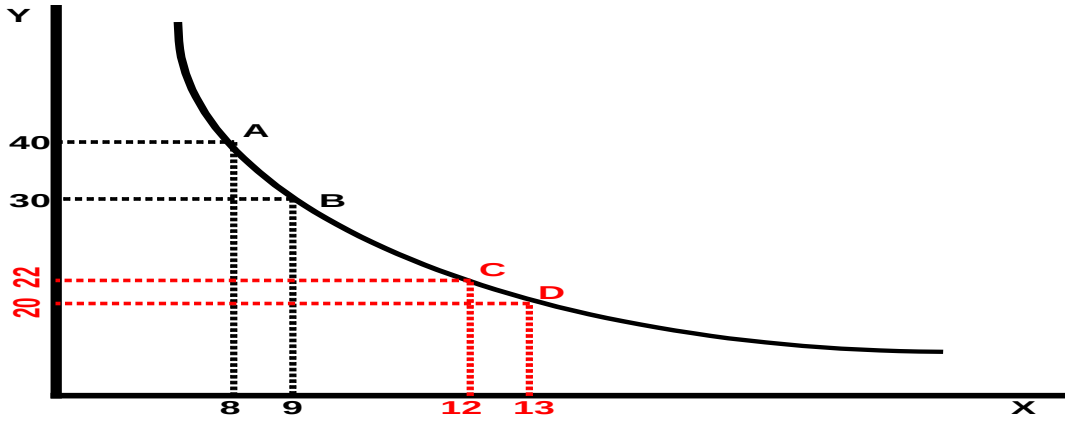



**Interpreting the Indifference Curves**

Interpreting/reading indifference curves takes us back to that third assumption of diminishing marginal rate of substitution. ***What is the marginal rate of substitution?***

This is the size of the reduction in the variable on the vertical axis that leaves the individual indifferent following an increase of one unit of the variable on the horizontal axis.

This is much clearer if we do it on a graph.



Another way of saying it is that **the MRS is the maximum amount a person will give up to obtain one more unit.** The maximum amount is the amount that leaves him just indifferent.

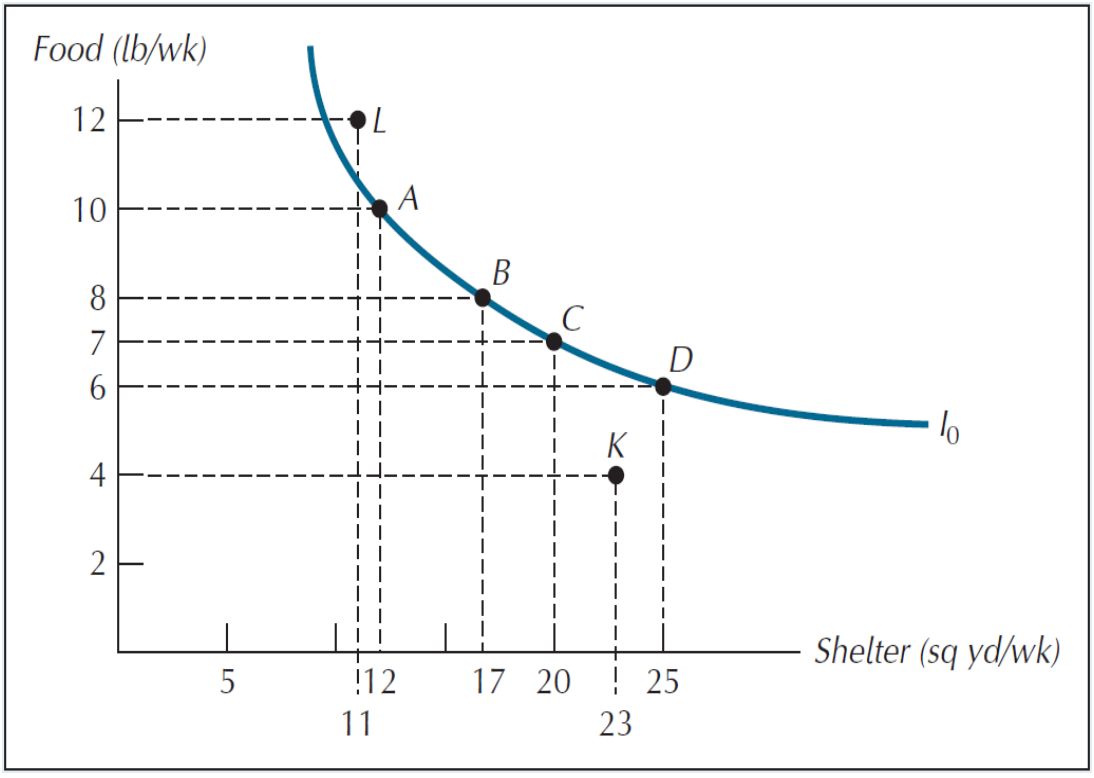
This should ring a bell in regard to demand: remember how we interpreted the height of a demand curve. It was the maximum amount a person would pay – in effect, how much he valued that unit. Well, we'll link all this up.

**So this is what we mean by Marginal Rate of Substitution – *but what about this diminishing part?***

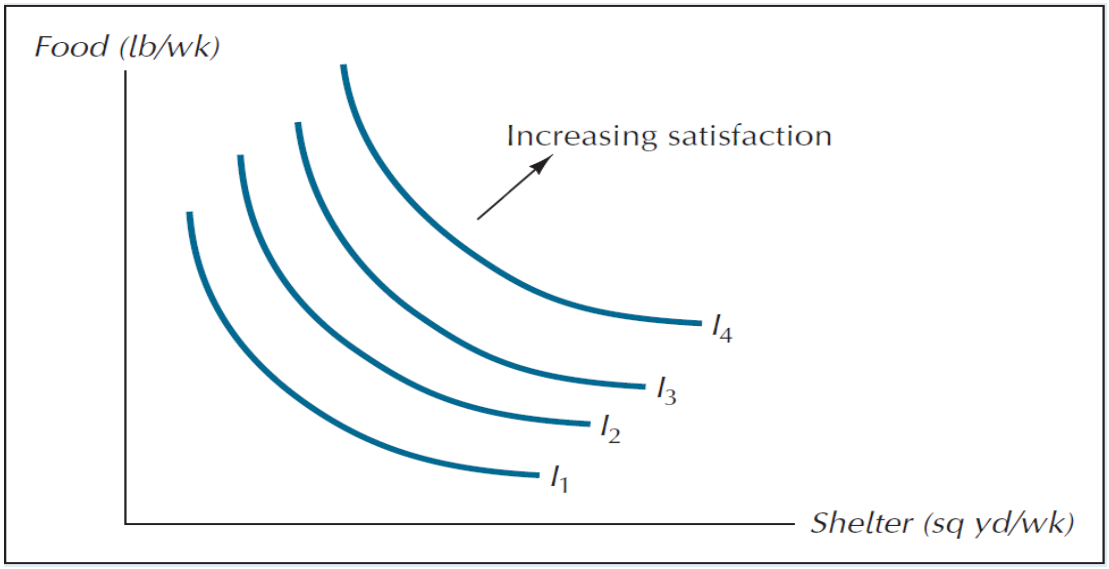
The diminishing part comes from what happens as this individual consumes more and more of X. As you can see from the slope, it is getting flatter. *What this means is that as he has more X, the maximum amount of Y he is willing to give up to get more X gets less and less.*

Up at the top (steep slope), he is willing to give up 10Y (go from 40Y to 30Y) to get one more X (from bundle A to bundle B) but down here, he is only willing to give up 2Y to get one more X (from bundle **C** to bundle D). The rate I am willing to substitute in X for Y is diminishing as I consume more X.

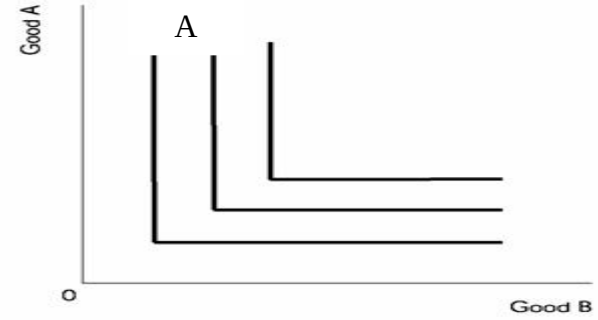
**A good example of an indifference curve**

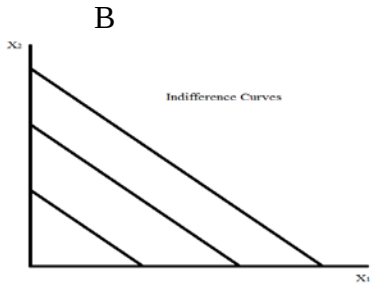


**Indifference Map**




Other Examples - How would you interpret indifference curves that looked like this?




Look at A. In this case you must consume these two goods in fixed ratios – like one left and one right shoe, or one set of eye glass frames and one set of lenses – things like that.

When looking at the graph you can see that having one right shoe and two left shoes that the additional left shoe doesn't make you any better off.

Look at B - These are perfect substitutes

**The Slope of the Indifference Curve**

Before we get on to the budget constraint, we talk about utility and put the indifference curve into a utility framework. Economists assume that individuals maximize utility -- which is some measure of personal welfare or satisfaction. In other words, we all have some function of U = *U*(X, Y), all this says is that my consumption of X and Y increases my satisfaction.

Marginal utility is simply: $MU_X = \frac{\Delta U}{\Delta X} = \frac{dU}{dX} = \frac{\partial U}{\partial X}$.

Along an indifference curve, we say that **utility is constant** -- in other words, if I am indifferent between two bundles that means it gives me the same level of utility.

Therefore we have the following: $MU_X\Delta X + MU_Y\Delta Y = 0$

The first terms is the addition in utility resulting from additional X (so $\Delta X>0$); the second term is the decrease in utility resulting from the decrease in Y (so $\Delta Y<0$).

Rearranging $MU_X\Delta X = -MU_Y\Delta Y$.

Remember, marginal rate of substitution is the slope of the indifference curve, which is $\Delta Y/\Delta X$ (rise over the run). So as you can see $MRS = \frac{\Delta Y}{\Delta X} = -\frac{MU_X}{MU_Y}$. **So the marginal rate of substitution is simply the ratio of the marginal utilities.**

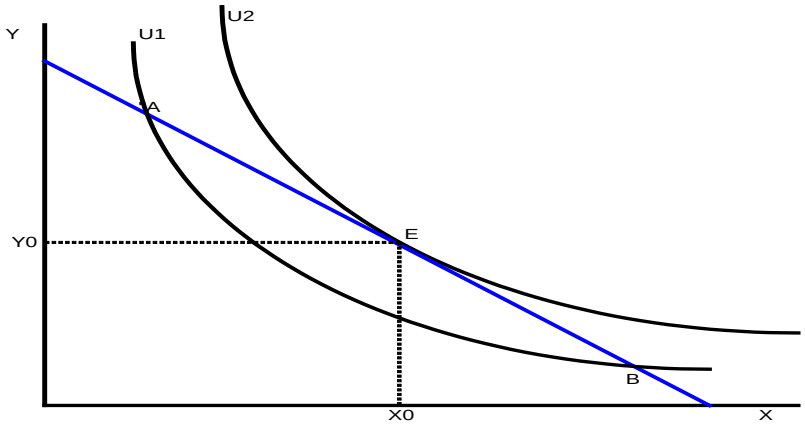
$U = U(X,Y)$ *Taking total differential*

$$dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy = 0 \qquad MU_X\,dx + MU_Y\,dy = 0 \qquad \frac{dy}{dx} = -\frac{MU_X}{MU_Y}$$

Understanding this will come in handy later.

**Consumer Equilibrium**

Now it's time to bring these two concepts together and build a model of human behavior. Look at the following graph:




At point A, what is the slope of the indifference curve? Remember the slope is the marginal rate of substitution that tells us what the **maximum amount of Y the person is willing to give up to obtain one more unit of X** -- let's say the slope is 10.

At point A, what is the slope of the budget constraint? Remember, the slope is the relative price of X -- in other words, **what do I have to pay**? Let's say it is 2 -- which means that to obtain one more unit of X, I give up two units of Y.

Now, if I am willing to give up 10 but only have to give up 2, what will happen to my utility -- clearly I am better off and therefore, by definition, I will move up a higher indifference curve.

So if we move up a higher indifference curve, we go through the same decision process -- **is what I'm willing to pay greater than what I have to pay?** -- if so, I buy it and am better off as a result -- this is identical to what we did with a demand curve -- in fact, later this week we will derive a demand curve from this graph.

Now look at point E -- at this point, the indifference curve is tangent to the budget line. At the point of tangency, what can you say about the slopes? By definition, they are equal. So what does that say about this person? Well the maximum amount I am willing to pay is equal to the amount I have to pay -- i.e. MRS is to the price ratio.

If I go beyond that, then the maximum amount I'm willing to pay is less than what I have to pay and therefore, if I purchase more, I make myself worse off.

So point E gives us is one point on the demand curve. For a given income, price of X, and price of Y, and given my preferences this is how much X (and Y) I will consume.

So E is the equilibrium point -- that is where we will always be (or get to very quickly).

Remember we are doing comparative statics -- which is just the comparison of different equilibrium points.

At point E, the slopes are equal.

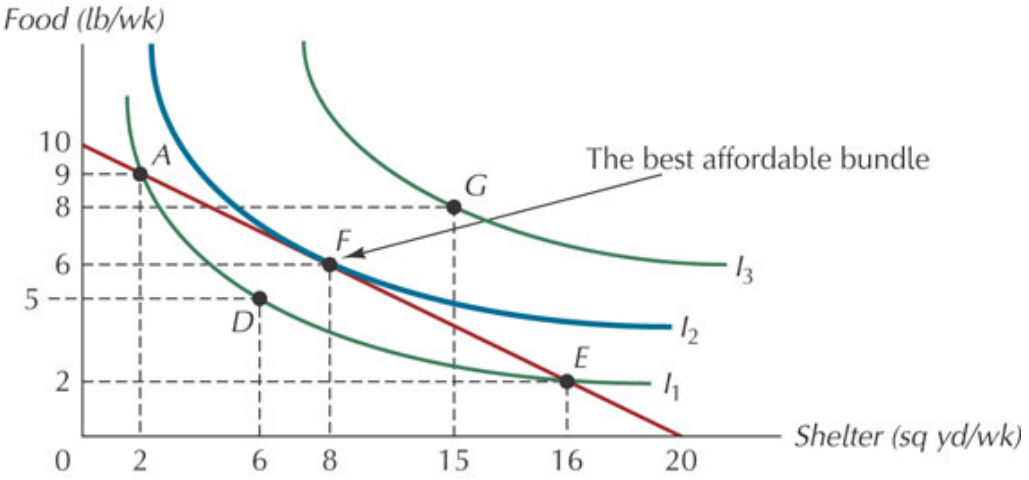
The slope of the indifference curve is the marginal rate of substitution:

$MRS = -\frac{MU_X}{MU_Y}$ The slope of the budget constraint is the price ratio: $-\frac{P_X}{P_Y}$.

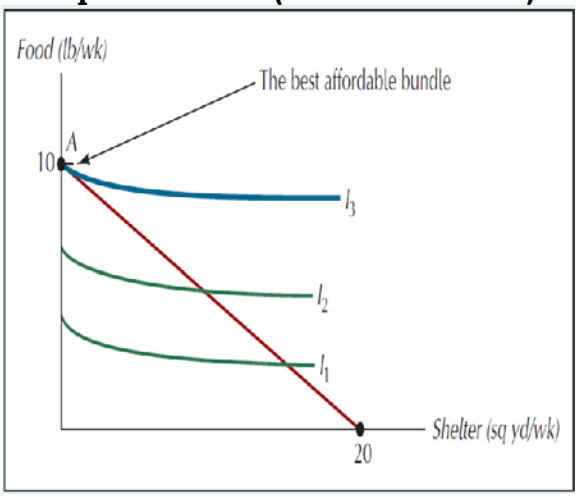
Therefore, at point E the following is true: $\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$ or $\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$

The economic interpretation of this is that at equilibrium the utility per dollar spent must be equal across all goods. In other words, if I can take a dollar away from Y and spend it on X and get more utility, I will do that (and vice versa).

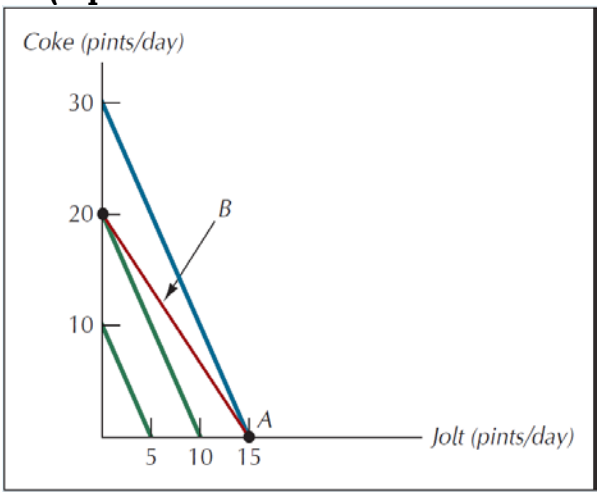
## Another Example



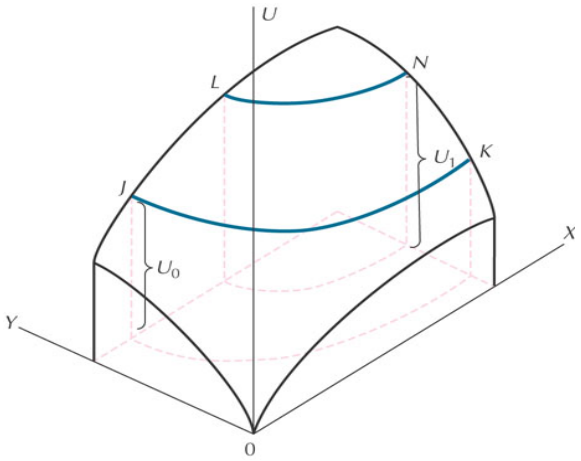

## Some Special Cases (Corner Solution)

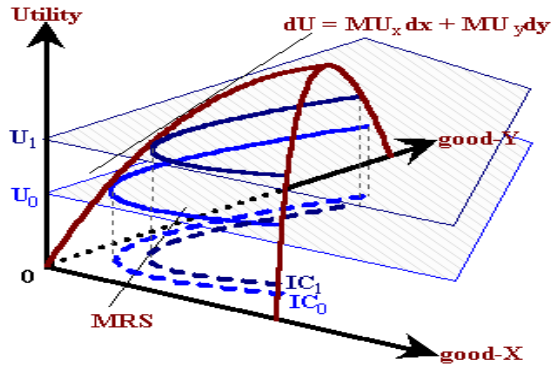



## (Equilibrium with Perfect Substitutes)




## Three- Dimensional Utility Curve



**Example: Let Mary's weekly expenditure be \$10.00 and $P_N$ = \$1.00 and $P_M$= \$2.00**
**N = newspapers, M= magizines**

| N | U(N) | MU(N) | MU(N)/PN | M | U(M) | MU(M) | MU(M)/PM |
|---|---|---|---|---|---|---|---|
| 0 | 0 | | | 0 | 0 | | |
| | | 12 | 12 | | | 20 | 10 |
| 1 | 12 | | | 1 | 20 | | |
| | | 8 | 8 | | | 12 | 6 |
| 2 | 20 | | | 2 | 32 | | |
| | | 6 | 6 | | | 8 | 4 |
| 3 | 26 | | | 3 | 40 | | |
| | | 4 | 4 | | | 4 | 2 |
| 4 | 30 | | | 4 | 44 | | |
| | | 2 | 2 | | | 2 | 1 |
| 5 | 32 | | | 5 | 46 | | |

**Prove Mary will buy 4 Newspaper and 3 Magazines**

**CASH OR FOOD STAMPS?**

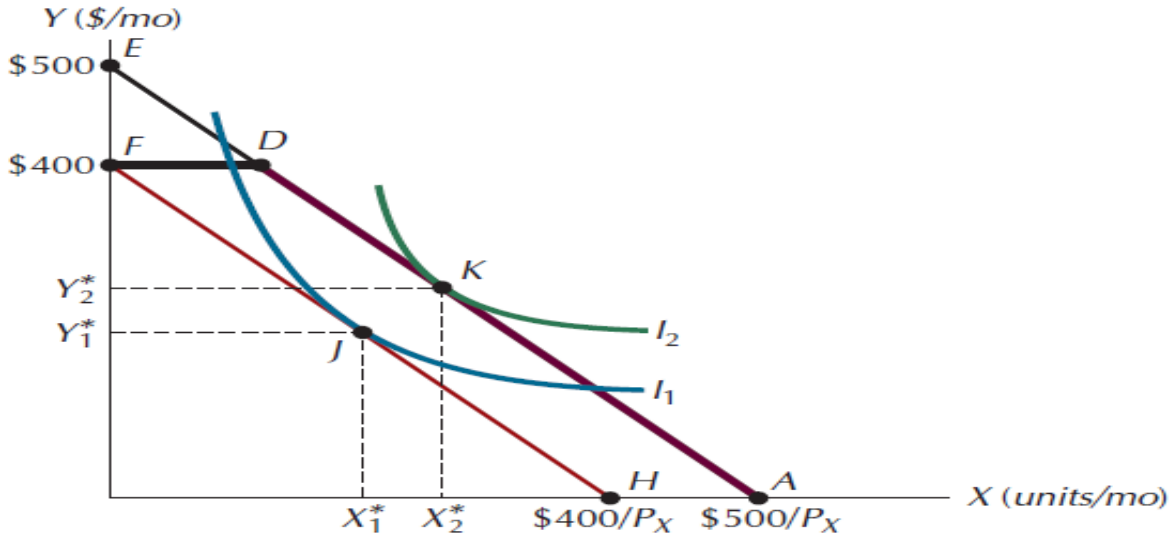
Food Stamp Program has following objective - to alleviate hunger.

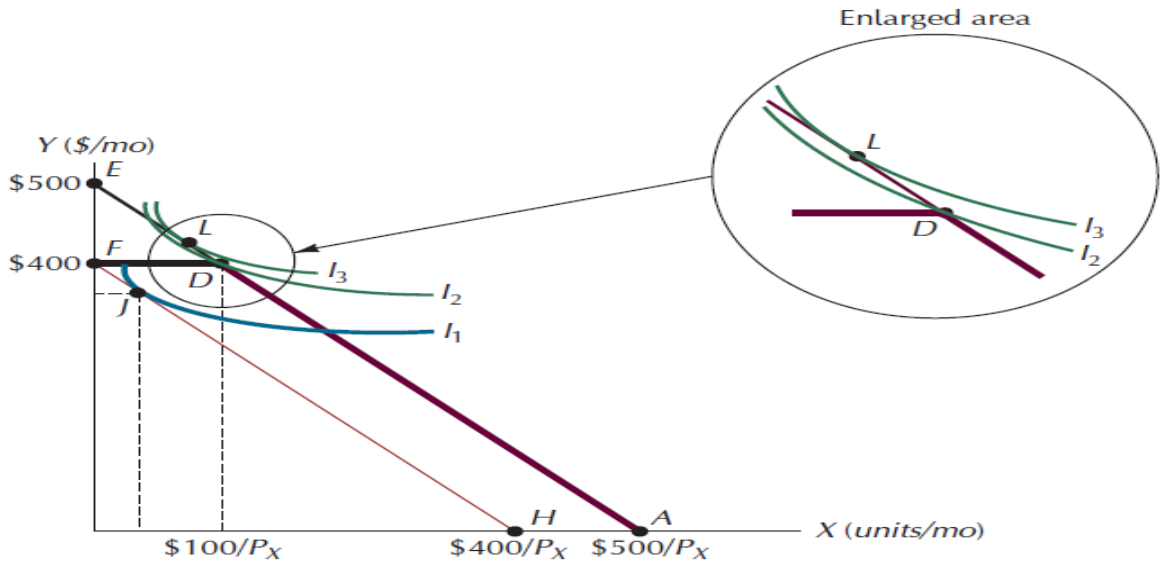
How does it work?

People whose incomes fall below a certain level are eligible to receive a specified quantity of food stamps.

Stamps cannot be used to purchase cigarettes, alcohol, and various other items.

The government gives food retailers cash for the stamps they accept.






**MATHEMATICAL ANALYSIS OF CONSUMER BEHAVIOR**

**1. The Indifference Curve**

The total utility function is given as: $U(X, Y)$.

So that an indifference curve is given by: $U(X, Y) = C$,

where "C" is a constant level of utility along the stable indifference curve.

Take the total differential of this indifference curve and set it equal to zero (Why?):

U = U(X, Y) $$dU = \frac{\partial U}{\partial X}dX + \frac{\partial U}{\partial Y}dY = 0 \quad \text{or} \quad \frac{\partial U}{\partial X}dX + \frac{\partial U}{\partial Y}dY = 0$$

(**Note**: By taking the total differential of an indifference curve and setting it equal to zero, both $X$ and $Y$ are allowed to change simultaneously, but there is no change in total utility.)

We note that some texts define the MRS as the negative of the slope. If that is the case, solving for the negative of the slope of the indifference curve, $-dY/dX$:

$$-\frac{dY}{dX} = MRS_{X,Y} = \frac{\frac{\partial U}{\partial X}}{\frac{\partial U}{\partial Y}} \quad \text{or} \quad MRS_{X,Y} = \frac{MU_X}{MU_Y}$$

The slope of an indifference curve is the $MRS_{X,Y}$; which is equal to the ratio of the marginal utility of the two goods consumed.

**2. The Budget Line**

**Budget Line** (or **Income Constraint**)

The locus of budgets (total expenditures on alternative combinations of goods) that can be purchased if all money income is spent. Algebraically:

$$M = P_x X + P_y Y,$$

Where M is total money income, $P_X$ is the nominal price of good *X*, $P_Y$ is the nominal price of good *Y*, and *X* and *Y* are the quantities of both goods purchased.

Solving for Y: $Y = \frac{M}{P_Y} - \frac{P_X}{P_Y} X$ (**Note**: The slope of the budget line is the ratios of the price of X to Y.) Slope of Budget Line = $-\frac{P_X}{P_Y}$

If M = *$200*, $P_X$ is *$20*, and $P_Y$ is *$10*, then the budget line or income constraint is:

$$Y = \frac{\$200}{\$10} - \frac{\$20}{\$10} X \quad \text{or} \quad Y = 20 - 2X$$

Thus: $-\frac{P_X}{P_Y} = -\frac{20}{10} = -2$

We often ignore the negative sign.

**3. Consumer Equilibrium**

A consumer maximizes his/her total utility from consumption subject to the budget constraint. Consumer equilibrium implies:

1. The consumer's highest indifference curve is tangent to the budget line. (Why?)
2. Equality between the $MRS_{X,Y}$ and the relative price ratio of good *X*. (Why?)
3. The ratios of the *MU* to the price are equal for all goods. (Why?)

(Consumer allocates expenditures so that the utility of the last dollar spent on each good is equal.)

The point of consumer equilibrium is where: $MRS_{XforY} = \frac{P_X}{P_Y}$ or $\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$

Cross-multiplying: $\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$

**Formal Mathematical Proof of Consumer Equilibrium**

Maximize: $U = U(X, Y)$ Subject to: $M = P_x X + P_y Y$,

(**Note**: A consumer maximizes his/her total utility given the budget constraint.)

This is a constrained maximization problem, hence, the Lagrangean function is:

$$L = U(X,Y) + \lambda(M - P_X X - P_Y Y)$$

Taking the derivative of $L$ with respect to each good (i.e., $X$ and $Y$) and setting it equal to zero:

$$\frac{\partial L}{\partial X} = \frac{\partial U}{\partial X} - \lambda P_X = 0 \qquad \frac{\partial L}{\partial Y} = \frac{\partial U}{\partial Y} - \lambda P_Y = 0$$

Solving for $\lambda$: $\lambda = \frac{MU_X}{P_X}$ and $\lambda = \frac{MU_Y}{P_Y}$

Equating these two equations: $\frac{MU_X}{P_X} = \frac{MU_Y}{P_y}$

Consumer equilibrium requires that the ratio of the marginal utility and the money price of each good be equalized for all goods consumed.

Alternatively stated: $\frac{MU_X}{MU_Y} = \frac{P_X}{P_y}$

Consumer equilibrium requires that the ratio of the marginal utilities of the goods equal the relative price ratio.

**Example:**

An individual consumes products X and Y and spends $24 per time period. The prices of the two goods are $3 per unit for X and $2 per unit for Y. The consumer in this case has a utility function expressed as:

$$U = X^2 Y^2$$

a. Determine the values of X and Y that will maximize utility in the consumption of X and Y.

b. Determine the total utility that will be generated per unit of time for this individual.

**Problem:**

Consider the utility function $U = X^{\alpha}Y^{\beta}$ What is the MRS?

**Homothetic Preferences -** If the *MRS* depends only on the ratio of the amounts of the two goods, not on the quantities of the goods, the utility function is homothetic

We can show using the Cobb – Douglas function.

$$MRS = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = \frac{\alpha x^{\alpha-1} y^{\beta}}{\beta x^{\alpha} y^{\beta-1}} = \frac{\alpha}{\beta} \cdot \frac{y}{x}$$