

Chapter 5 Lecture - Discounted Cash Flow Valuation

Chapter 5 Lecture - Discounted Cash Flow Valuation



Learning Objectives

After studying this chapter, you should be able to:

- **LO1** Determine the future and present value of investments with multiple cash flows.
- **LO2** Calculate loan payments and find the interest rate on a loan.
- **LO3** Describe how loans are amortized or paid off.
- **LO4** Explain how interest rates are quoted (and misquoted).

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Future Value: Multiple Cash Flows (1)

- You think you will be able to deposit \$4,000 at the end of each of the next three years in a bank account paying 8 percent interest.
- You currently have \$7,000 in the account.
- How much will you have in 3 years?
- How much in 4 years?

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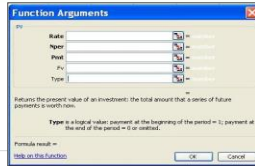
Future Value: Multiple Cash Flows (1)

- Find the value at year 3 of each cash flow and add them together.
 - Year 0: $FV = \$7,000(1.08)^3 = \$ 8,817.98$
 - Year 1: $FV = \$4,000(1.08)^2 = \$ 4,665.60$
 - Year 2: $FV = \$4,000(1.08)^1 = \$ 4,320.00$
 - Year 3: value = $\$ 4,000.00$
 - Total value in 3 years = $\$21,803.58$
- Value at year 4 = $\$21,803.58(1.08) = \$23,547.87$

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Chapter 5 Lecture - Discounted Cash Flow Valuation

FV Excel Solution



Excel Solution					
Year	Nper	Rate	PV	PMT	FV
0	3	0.08	-7000	0	8,817.98
1	2	0.08	-4000	0	4,665.60
2	1	0.08	-4000	0	4,320.00
3					4,000.00
Value at year 4:					21,803.58
Year	Nper	Rate	PV	PMT	FV
4	1	0.08	-21,803.58	0	23,547.87

=FV(Rate, Nper, PMT, PV)

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FV Example Calculator Solution

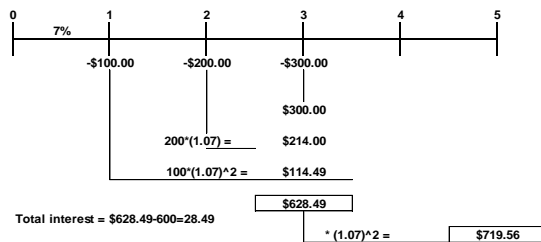
Calculator Solution					
Year	N	I/Y	PV	PMT	CPT FV
0	3	8	-7000	0	8,817.98
1	2	8	-4000	0	4,665.60
2	1	8	-4000	0	4,320.00
3					4,000.00
Value at year 4:					21,803.58
Year	N	I/Y	PV	PMT	CPT FV
4	1	8	-21,803.58	0	23,547.87

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Future Value: Multiple Cash Flows (2)

- If you deposit \$100 in one year, \$200 in two years and \$300 in three years.
- How much will you have in three years at 7 percent interest?

TIMELINE



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Future Value: Multiple Cash Flows (2)

- If you deposit \$100 in one year, \$200 in two years and \$300 in three years.
- How much will you have in three years at 7 percent interest?
- How much in five years if you don't add additional amounts?
 - Year 1 CF: 2 N; -100 PV; 7 I/Y; CPT FV = 114.49
 - Year 2 CF: 1 N; -200 PV; 7 I/Y; CPT FV = 214.00
 - Year 3 CF: 0 N; -300 PV; 7 I/Y; CPT FV = 300.00
 - Total FV₃ = 628.49
 - Total FV₅ = $628.49 \times (1.07)^2 = 719.56$

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Future Value: Multiple Cash Flows (2)

Excel Solution

Rate	7%			
Year	Nper	CF	FV	Function
1	2	-100	\$114.49	=FV(0.07,2,0,-100)
2	1	-200	\$214.00	=FV(0.07,1,0,-200)
3	0	-300	\$300.00	=FV(0.07,0,0,-300)
Total FV at Year 3			\$628.49	
Total FV at Year 5			\$719.56	=(628.49)*(1.07)^2

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Future Value: Multiple Cash Flows (3)

- Suppose you invest \$500 in a mutual fund today and \$600 in one year.
- If the fund pays 9% annually, how much will you have in two years?

$$FV = \$500 \times (1.09)^2 = \$594.05$$

$$+ \$600 \times (1.09) = \underline{\$654.00}$$

$$= \$1,248.05$$

- How much will you have in 5 years if you make no further deposits?

$$FV = \$500(1.09)^5 + \$600(1.09)^4 = \$1,616.26$$

- Second way - use value at year 2:
- $FV = \$1,248.05(1.09)^3 = \$1,616.26$

Excel Solution

Year	Nper	Rate	PV	PMT	FV
0	2	0.09	-500	0	=594.05
1	1	0.09	-600	0	=654.00
Value at year 4:					=1,248.05
Year	Nper	Rate	PV	PMT	FV
5	3	0.09	-1,248.05	0	=1,616.26

=FV(Rate, Nper, PMT, PV)

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FV Example Calculator Solution

Calculator Solution						
Year	N	I/Y	PV	PMT	CPT	FV
0	2	9	500	0	594.05	
1	1	9	600	0	654.00	
						1,248.05
Value at year 4:						
Year	N	I/Y	PV	PMT	CPT	FV
5	3	9	1,248.05	0	1,616.26	
or						
Year	N	I/Y	PV	PMT	CPT	FV
0	5	9	500	0	769.31	
1	4	9	600	0	846.95	
						1,616.26

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Future Value: Multiple Cash Flows Another Example - Formula

- Suppose you plan to deposit \$100 into an account in one year and \$300 into the account in three years.
- How much will be in the account in five years if the interest rate is 8%?

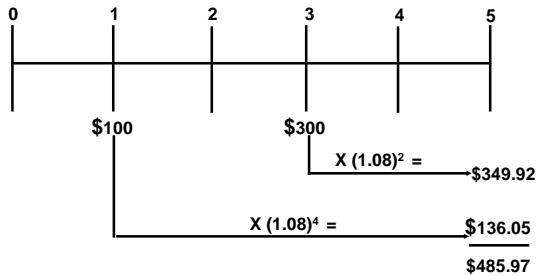
$$FV = \$100(1.08)^4 + \$300(1.08)^2 = \$136.05 +$$

$$\$349.92 = \$485.97$$

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Chapter 5 Lecture - Discounted Cash Flow Valuation

Example Time Line



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FV Calculator and Excel Solution

Calculator Solution						CPT
Year	N	I/Y	PV	PMT	FV	
1	4	8	-100	0	136.05	
3	2	8	-300	0	349.92	
					485.97	
Excel Solution						FV
Year	Nper	Rate	PV	PMT		
1	4	0.08	-100	0	136.05	
3	2	0.08	-300	0	349.92	
					485.97	
=FV(Rate, Nper, PMT, PV)						

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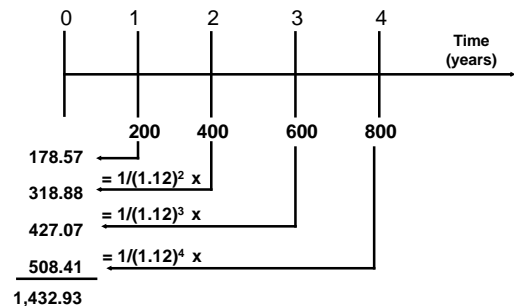
Present Value: Multiple Cash Flows

Find the PV of each cash flow and add them:

- Year 1 CF: $\$200 / (1.12)^1 = \$ 178.57$
- Year 2 CF: $\$400 / (1.12)^2 = \$ 318.88$
- Year 3 CF: $\$600 / (1.12)^3 = \$ 427.07$
- Year 4 CF: $\$800 / (1.12)^4 = \$ 508.41$
- Total PV = $\$1,432.93$

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Time Line

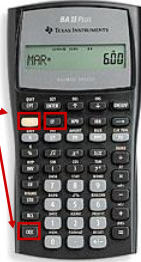


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Chapter 5 Lecture - Discounted Cash Flow Valuation

Multiple Uneven Cash Flows Using the TI BAI+ Cash Flow Worksheet

- Clear all:
 - Press *CF*
 - Then *2nd*
 - Then *CE/C*
- CF_0 is displayed as 0.00
- Enter the Period 0 cash flow
 - If an outflow, press +/- to change the sign
- To enter the figure in the cash flow register, press *ENTER*



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TI BAI+: Uneven Cash Flows

- Press the down arrow to move to the next cash flow register
- Enter the cash flow amount, press *ENTER* and the down arrow to move to the cash flow counter (*FOn*)
- The default counter value is "1"
 - To accept the value of "1", press the down arrow again
 - To change the counter, enter the correct count, press *ENTER* and then the down arrow



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TI BAI+: Uneven Cash Flows

- Repeat for all cash flows, in order.
- To find NPV:
 - Press *NPV*: *I* appears on the screen.
 - Enter the interest rate, press *ENTER*, and then the down arrow to display NPV.
 - Press *CPT*.



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TI BAI+: Uneven Cash Flows

Cash Flows:	Display	You Enter
CF0 = 0	C00	0 ENTER
CF1 = 200	C01	200 ENTER
CF2 = 400	F01	1 ENTER
CF3 = 600	C02	400 ENTER
CF4 = 800	F02	1 ENTER
	C03	600 ENTER
	F03	1 ENTER
	C04	800 ENTER
	F04	1 ENTER NPV
	I	12 ENTER down
	NPV	CPT
	1432.93	

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Chapter 5 Lecture - Discounted Cash Flow Valuation

PV of Multiple Uneven CFs – Calculator and Excel Solutions

Excel Solution

Rate	12%			
Period	Cash Flow	Present Value	Formula	
1	\$ 200.00	(\$178.57)	=PV(\$B\$1,A3,0,B3)	
2	\$ 400.00	(\$318.88)	=PV(\$B\$1,A4,0,B4)	
3	\$ 600.00	(\$427.07)	=PV(\$B\$1,A5,0,B5)	
4	\$ 800.00	(\$508.41)	=PV(\$B\$1,A6,0,B6)	
Total PV =		(\$1,432.93)	=SUM(C3:C6)	
		(\$1,432.93)	=NPV(B1,B3:B6)	

The functions require a PMT = 0.

Calculator Solution

Year	N	I/Y	FV	PMT	CPT PV
1	1	12	200	0	178.57
2	2	12	400	0	318.88
3	3	12	600	0	427.07
4	4	12	800	0	508.41
					1,432.93

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Present Value: Multiple Cash Flows Another Example – Formula Solution

- You are considering an investment that will pay you \$1,000 in one year, \$2,000 in two years and \$3,000 in three years.
- If you want to earn 10% on your money, how much would you be willing to pay?
 - $PV = \$1,000 / (1.1)^1 = \$ 909.09$
 - $PV = \$2,000 / (1.1)^2 = \$1,652.89$
 - $PV = \$3,000 / (1.1)^3 = \$2,253.94$
 - PV = \$4,815.92**

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Multiple Cash Flows – PV Example with Excel and Calculator Solutions

Calculator Solution						CPT PV
Year	N	I/Y	FV	PMT		
1	1	10	-1000	0		909.09
2	2	10	-2000	0		1,652.89
3	3	10	-3000	0		2,253.94
						4,815.92
Excel Solution						PV
Year	Nper	Rate	FV	PMT		
1	1	0.10	-1000	0		909.09
2	2	0.10	-2000	0		1,652.89
3	3	0.10	-3000	0		2,253.94
						4,815.92
=PV(Rate, Nper, PMT, FV)						

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Present Value: Multiple Cash Flows Another Example – Formula Solution

- You are considering an investment that will pay you \$1,000 in one year, \$2,000 in two years and \$3,000 in three years.
- If you want to earn 10% on your money, how much would you be willing to pay?
 - $PV = \$1,000 / (1.1)^1 = \$ 909.09$
 - $PV = \$2,000 / (1.1)^2 = \$1,652.89$
 - $PV = \$3,000 / (1.1)^3 = \$2,253.94$
 - PV = \$4,815.92**

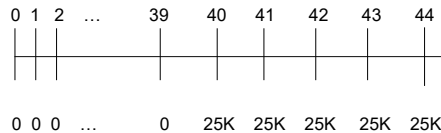
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Chapter 5 Lecture - Discounted Cash Flow Valuation

Saving For Retirement

You are offered the opportunity to put some money away for retirement. You will receive five annual payments of \$25,000 each beginning in 40 years. How much would you be willing to invest today if you desire an interest rate of 12%?

Timeline



Notice that the year 0 cash flow = 0 ($CF_0 = 0$)

Cash flows years 1–39 = 0 Cash flows years 40–44 = 25,000

Show the answer is **\$1,084.71**

5-25

Excel and Calculator Solution

Nper	0-39	0	Rate	0.12	
Nper	40	-25000			\$268.67
Nper	41	-25000			\$239.88
Nper	42	-25000			\$214.18
Nper	43	-25000			\$191.23
Nper	44	-25000			\$170.74
			PV=		\$1,084.71

Use cash flow keys:
 CF
 2nd CE/C
 CF0 0 ENTER
 C01 0 ENTER
 F01 39 ENTER
 C02 25000 ENTER
 F02 5 ENTER
 NPV
 I 12 ENTER
 DOWN
 CPT
 1084.71

5-26

Decisions, Decisions

- Your broker calls you and tells you that he has this great investment opportunity.
- If you invest \$100 today, you will receive \$40 in one year and \$75 in two years.
- If you require a 15% return on investments of this risk, should you take the investment?

Excel Solution

Use Formula $PV(Rate, Nper, Pmt, FV)$

$PV = PV(0.15, 1, 0, 40) = \34.78

$+ PV = PV(0.15, 2, 0, 75) = \56.71

= \$91.49

Don't take the investment - broker is charging more than you would be willing to pay.

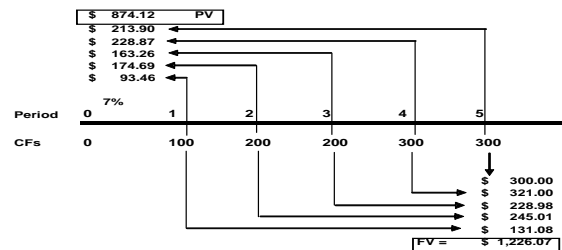
Use cash flow keys:

CF
 2nd CE/C
 CF0 0 ENTER
 C01 40 ENTER
 F01 1 ENTER
 C02 75 ENTER
 F02 2 ENTER
 NPV
 I 15 ENTER
 DOWN CPT
 91.49

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Quick Quiz

- Suppose you are looking at the following possible cash flows:
 - Year 1 CF = \$100;
 - Years 2 and 3 CFs = \$200;
 - Years 4 and 5 CFs = \$300.
 - The required discount rate is 7%.
- What is the value of the CFs at year 5?
- What is the value of the CFs today?



Chapter 5 Lecture - Discounted Cash Flow Valuation

Quick Quiz – Excel Solution

Year	Rate	7%		
Year	Nper	CF	PV	Formula
1	1	100	\$93.46	=-PV(\$C\$2,A4,0,C4)
2	2	200	\$174.69	=-PV(\$C\$2,A5,0,C5)
3	3	200	\$163.26	=-PV(\$C\$2,A6,0,C6)
4	4	300	\$228.87	=-PV(\$C\$2,A7,0,C7)
5	5	300	\$213.90	=-PV(\$C\$2,A8,0,C8)
		Total PV	\$874.17	=SUM(C4:C8)
Year	Nper	CF	FV	Year
1	4	100	\$131.08	=-FV(\$C\$2,B12,0,C12)
2	3	200	\$245.01	=-FV(\$C\$2,B13,0,C13)
3	2	200	\$228.98	=-FV(\$C\$2,B14,0,C14)
4	1	300	\$321.00	=-FV(\$C\$2,B15,0,C15)
5	0	300	\$300.00	=-FV(\$C\$2,B16,0,C16)
		Total FV	\$1,226.07	=SUM(C12:C16)

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Annuities and Perpetuities

- Annuity – finite series of equal payments that occur at regular intervals
 - If the first payment occurs at the end of the period, it is called an ordinary annuity
 - If the first payment occurs at the beginning of the period, it is called an annuity due
- Perpetuity – infinite series of equal payments.

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Annuities and Perpetuities Basic Formulas

- Perpetuity: $PV = PMT / r$
- Annuities:

$$PV = PMT \left[\frac{1 - \frac{1}{(1+r)^t}}{r} \right]$$

$$FV = PMT \left[\frac{(1+r)^t - 1}{r} \right]$$

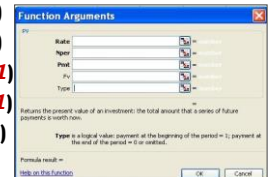
Derivation of Annuity Formula

<http://www.ftsmo.du.edu/public/texts/bondtutor/appendix1a.htm>

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Excel Spreadsheet Functions

- $FV(Rate, Nper, PMt, PV, 0/1)$
- $PV(Rate, Nper, PMt, FV, 0/1)$
- $RATE(Nper, PMt, PV, FV, 0/1)$
- $NPER(Rate, PMt, PV, FV, 0/1)$
- $PMT(Rate, Nper, PV, FV, 0/1)$



- Inside parenthesis: $(RATE, NPER, PMT, PV, FV, 0/1)$
- “0/1” Ordinary annuity = 0 (default; no entry needed)
- Annuity Due = 1 (must be entered)

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Chapter 5 Lecture - Discounted Cash Flow Valuation

Important Points to Remember

- Interest rate and time period must match!
 - Annual periods \Rightarrow annual rate
 - Monthly periods \Rightarrow monthly rate
- The Sign Convention
 - Cash inflows are positive
 - Cash outflows are negative

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Sign Convention Example Calculator

5 N
10 I/Y
-100 PV
20 PMT
CPT FV = \$38.95

Implies you deposited \$100 today and plan to **WITHDRAW** \$20 a year for 5 years

+CF = Cash INFLOW to YOU

5 N
10 I/Y
-100 PV
-20 PMT
CPT FV = \$283.15

Implies you deposited \$100 today and plan to **ADD** \$20 a year for 5 years

-CF = Cash OUTFLOW from you

5-34

Annuity

- You can afford \$632 per month.
- Going rate = 1%/month for 48 months.
- How much can you borrow?
- You borrow money TODAY so you need to compute the **present value**.

48 N
1 I/Y
632 PMT
0 FV
CPT PV = 23,999.54 (\$24,000)

$$PV = 632 \left[\frac{1 - \frac{1}{(1.01)^{48}}}{.01} \right] = 23,999.54$$

Excel Solution

=PV(0.01,48,-632,0)

Rate	0.01		
Nper	48		
Pmt	-632		
		PV=	\$23,999.54

5-35

Annuity - Sweepstakes Example

- Suppose you win the Publishers Clearinghouse \$10 million sweepstakes.
- The money is paid in equal annual installments of \$333,333.33 over 30 years.
- If the appropriate discount rate is 5%, how much is the sweepstakes actually worth today?
 - $PV = \$333,333.33[1 - 1/1.05^{30}] / .05 = \$5,124,150.29$

Excel Solution

=PV(5, 30, 333333.33, 0)

Rate	0.05		
Nper	30		
Pmt	-333,333.33		
Type	0		
		PV =	\$5,124,150.29

N 30
I/Y 5
PMT \$333,333.33
FV 0
CPT PV
\$ (5,124,150.29)

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Chapter 5 Lecture - Discounted Cash Flow Valuation

Buying a House

- You are ready to buy a house and you have \$20,000 for a down payment and closing costs.
- Closing costs are estimated to be 4% of the loan value.
- You have an annual salary of \$36,000.
- The bank is willing to allow your monthly mortgage payment to be equal to 28% of your monthly income.
- The interest rate on the loan is 6% per year with monthly compounding (.5% per month) for a 30-year fixed rate loan.
- How much money will the bank loan you?
- How much can you offer for the house?

5-37

Buying a House – Calculator Solution

- **Bank loan**
 - Monthly income = $36,000 / 12 = 3,000$
 - Maximum payment = $.28(3,000) = 840$
 - 360 N (30*12)
 - 0.5 I/Y
 - -840 PMT
 - CPT PV = 140,105
- **Total Price**
 - Closing costs = $.04(140,105) = 5,604$
 - Down payment = $20,000 - 5604 = 14,396$
 - Total Price = $140,105 + 14,396 = 154,501$

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Buying a House - Continued

- **Bank loan**
 - Monthly income = $36,000 / 12 = 3,000$
 - Maximum payment = $.28(3,000) = 840$

Excel Solution

=PV(.005,360,-840,0)

Rate	.06/12	0.005		
Nper	12*30	360		
Pmt	-840	-840.00		
			Loan (PV)=	\$140,104.96

- **Total Price**
 - Closing costs = $.04(140,105) = 5,604$
 - Down payment = $20,000 - 5604 = 14,396$
 - Total Price = $140,105 + 14,396 = 154,501$

5-39

Quick Quiz

- You know the payment amount for a loan and you want to know how much was borrowed.
 - Do you compute a present value or a future value?
- **Example** - You want to receive \$5,000 per month in retirement. If you can earn .75% per month and you expect to need the income for 25 years, how much do you need to have in your account at retirement?

= -595,808.11

Excel Solution

=PV(0.0075,300,5000,0)

Rate	0.0075			
Nper	300			
Pmt	5000			
			Amount Needed (PV)	(\$595,808.11)

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Chapter 5 Lecture - Discounted Cash Flow Valuation

Finding the Payment

- Suppose you want to borrow \$20,000 for a new car.
- You can borrow at 8% per year, compounded monthly ($8/12 = .66667\%$ per month).
- If you take a 4 year loan, what is your monthly payment?

Excel Solution

=PMT(0.0066667,48,20000,0)

4(12) = 48 N
 0.66667 I/Y
 20,000 PV
 0 FV
 CPT PMT = -488.26

Rate	0.00666667		
Nper	48		
Pv	20000		
Monthly Payment (PMT)=		(\$488.26)	

5-41

Finding the Number of Payments Example

- \$1,000 due on credit card
- Payment = \$20 month minimum
- Rate = 1.5% per month
- The sign convention matters!!!

Excel Solution

=NPER(0.015,-20,1000,0)

1.5 I/Y
 1000 PV
 -20 PMT
 0 FV
 CPT N = 93.111 months
 = 7.75 years

Rate	0.015		
Pmt	-20		
Pv	1000		
	NPER =	93.11	(months)
	Years =	7.759254	

5-42

Finding the Number of Payments Another Example

Suppose you borrow \$2,000 at 5% and you are going to make annual payments of \$734.42. How long before you pay off the loan?

Excel Solution

=NPER(0.05,-734.42,2000,0)

Rate	0.05		
Pmt	-734.42		
Pv	2000		
	NPER=	2.999987	= 3 years

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Finding the Rate

Suppose you borrow \$10,000 from your parents to buy a car. You agree to pay \$207.58 per month for 60 months. What is the monthly interest rate?

Excel Solution

=RATE(60,-207.58,10000,0)

Nper	60		
Pmt	-207.58		
Pv	10000		
	RATE =	0.00749939	
		0.75% Per month	

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Chapter 5 Lecture - Discounted Cash Flow Valuation

Quick Quiz

You want to receive \$5,000 per month for the next 5 years. How much would you need to deposit today if you can earn .75% per month?

Excel Solution

`=PV(0.0075,60,5000,0)`

300 N Months
0.75 I/Y ← Monthly rate
5000 PMT ← Monthly Payment
0 FV
CPT PV = -595,808.11

Rate	0.0075		
Nper	60		
Pmt	5000		
	PV=		(\$240,866.87)

5-45

Quick Quiz

- You want to receive \$5,000 per month for the next 5 years.
- What monthly rate would you need to earn if you only have \$200,000 to deposit?

Excel Solution

`=RATE(60,5000,-200000,0)`

60 N
-200000 PV
5000 PMT
0 FV
CPT I/Y = 1.4395% per month

Nper	60		
Pmt	5000		
Pv	-200000		
		RATE=	0.014394781
		=	1.439% per month

5-46

Quick Quiz

- Suppose you have \$200,000 to deposit and can earn .75% per month.
 - How many months could you receive the \$5,000 payment?

Excel Solution

`=NPER(0.0075,5000,-200000,0)`

0.75 I/Y
-200000 PV
5000 PMT
0 FV
CPT N = 47.73 months
≈ 4 years

Rate	0.0075		
Pmt	-5000		
Pv	200000		
		NPER=	47.73477457
		=	48 months or 4 years

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Quick Quiz

- Suppose you have \$200,000 to deposit and can earn .75% per month.
- How much could you receive every month for 5 years?

Excel Solution

`=PMT(0.0075,60,-200000,0)`

60 N
0.75 I/Y
-200000 PV
0 FV
CPT PMT = 4151.67

Rate	0.0075		
Nper	60		
Pv	-200000		
		PMT=	\$4,151.67

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Chapter 5 Lecture - Discounted Cash Flow Valuation

Future Values for Annuities

Suppose you begin saving for your retirement by depositing \$2,000 per year in an IRA. If the interest rate is 7.5%, how much will you have in 40 years?

$$FV = PMT \left[\frac{(1+r)^t - 1}{r} \right]$$

$$FV = 2000 \left[\frac{(1.075)^{40} - 1}{.075} \right] = 454,513.04$$

5-49

Excel and Calculator Solution

Excel Solution

=FV(0.075,40,-2000,0)

Rate	0.075		
Nper	40		
Pmt	2000		
		FV=	(\$454,671.84)

40 N
7.5 I/Y
0 PV
-2000 PMT
CPT FV = 454513.04

5-50

Annuity Due

You are saving for a new house and you put \$10,000 per year in an account paying 8%. The first payment is made **today**. How much will you have at the end of 3 years?

$$FV_{AD} = PMT \left[\frac{(1+r)^t - 1}{r} \right] (1+r)$$

$$FV_{AD} = 10000 \left[\frac{(1.08)^3 - 1}{.08} \right] (1.08) = 35,061.12$$

5-51

Excel and Calculator Solution

Excel Solution

=FV(0.08,3,-10000,0,1)

Rate	0.08		
Nper	3		
Pmt	-10000		
Type	1	FV	\$35,061.12

2nd BGN 2nd SET
3 N
8 I/Y
0 PV
-10000 PMT
CPT FV = 35061.12
2nd BGN 2nd SET

5-52

Chapter 5 Lecture - Discounted Cash Flow Valuation

Quick Quiz

You want to have \$1 million to use for retirement in 35 years. If you can earn 1% per month, how much do you need to deposit on a monthly basis if the first payment is made in one month?

Ordinary Annuity

Excel Solution

=PMT(0.01,420,0,1000000)

420	N
1	I/Y
0	PV
1000000	FV
CPT PMT = -155.50	

Rate	0.01		
Nper	420		
FV	1000000		
Type	0	PMT=	(\$155.50)

5-53

Quick Quiz

You want to have \$1 million to use for retirement in 35 years. If you can earn 1% per month, how much do you need to deposit on a monthly basis if the first payment is made today?

Annuity Due

Excel Solution

=PMT(0.01,420,0,1000000,1)

Rate	0.01		
Nper	420		
FV	1000000		
Type	1	PMT=	(\$153.96)

2 nd BGN 2 nd SET	
420	N
1	I/Y
0	PV
1000000	FV
CPT PMT = -153.96	
2 nd BGN 2 nd SET	

5-54

Summary of Calculations

I. Symbols

PV = Present value, what future cash flows are worth today
 FV_t = Future value, what cash flows are worth in the future at time *t*
r = Interest rate, rate of return, or discount rate per period—typically, but not always, one year
t = Number of periods—typically, but not always, the number of years
C = Cash amount

II. Future value of *C* invested per period for *t* periods at *r* percent per period

$FV_t = C \times [(1 + r)^t - 1]/r$
 A series of identical cash flows paid for a set number of periods is called an annuity, and the term $[(1 + r)^t - 1]/r$ is called the *annuity future value factor*.

III. Present value of *C* per period for *t* periods at *r* percent per period

$PV = C \times [1 - 1/(1 + r)^t]/r$
 The term $[1 - 1/(1 + r)^t]/r$ is called the *annuity present value factor*.

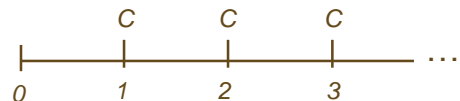
IV. Present value of a perpetuity of *C* per period

$PV = C/r$
 A perpetuity has the same cash flow every period forever.

5-55

Perpetuity

A constant stream of cash flows that lasts forever



$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \dots$$

$$PV = \frac{C}{r}$$

5-56

Chapter 5 Lecture - Discounted Cash Flow Valuation

Perpetuity: Example

What is the value of a British consol that promises to pay £15 every year for ever? The interest rate is 10-percent.

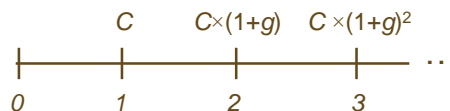


$$PV = \frac{\pounds 15}{.10} = \pounds 150$$

5-57

Growing Perpetuity

A growing stream of cash flows that lasts forever



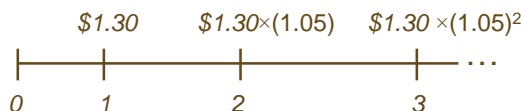
$$PV = \frac{C}{(1+r)} + \frac{C \times (1+g)}{(1+r)^2} + \frac{C \times (1+g)^2}{(1+r)^3} + \dots$$

$$PV = \frac{C}{r-g}$$

5-58

Growing Perpetuity: Example

- The expected dividend next year is \$1.30, and dividends are expected to grow at 5% forever.
- If the discount rate is 10%, what is the value of this promised dividend stream?



$$PV = \frac{\$1.30}{.10 - .05} = \$26.00$$

5-59

Quick Quiz

You are considering a preferred stock that pays a quarterly dividend of \$1.50. If your desired return is 3% per quarter, how much would you be willing to pay?

$$PV = \frac{C}{r}$$

$$PV = \frac{\$1.50}{.03} = \$50$$

5-60

Chapter 5 Lecture - Discounted Cash Flow Valuation

Interest Rates

- **Effective Annual Rate (EAR)**
 - The interest rate expressed as if it were compounded once per year.
 - Used to compare two alternative investments with different compounding periods
- **Annual Percentage Rate (APR) “Nominal”**
 - The annual rate quoted by law
 - $APR = \text{periodic rate} \times \text{number of periods per year}$
 - $\text{Periodic rate} = APR / \text{periods per year}$

5-61

Things to Remember

- You **ALWAYS** need to make sure that the interest rate and the time period match.
 - Annual periods → annual rate.
 - Monthly periods → monthly rate.
- If you have an APR based on monthly compounding, you have to use monthly periods for lump sums or adjust the interest rate accordingly.

5-62

EAR Formula

$$EAR = \left[1 + \frac{APR}{m} \right]^m - 1$$

APR = the quoted rate

m = number of compounds per year

5-63

EAR and APR in TI BA II+

2nd ICONV

2nd CE/C (to clear the memory)

• 3 fields in worksheet:

- NOM (Nominal rate-APR)
- EFF (Effective annual rate)
- C/Y (Compounding periods/yr)

– To compute EFF, enter the NOM and C/Y values, move to EFF and press CPT

– To compute NOM, enter the EFF and C/Y values, move to NOM and press CPT

5-64

Chapter 5 Lecture - Discounted Cash Flow Valuation

EAR and NOM in Excel

- 2 Functions:
 - =EFFECT(Nom, Nper)
 - =NOMINAL(Eff, Nper)
- Note Nominal is the same as APR
- All rates entered as decimals
- Nper = number of compounding periods per year

Reminder TOOLS ... Add-Ins ... ANALYSIS TOOLPAK

5-65

Decisions, Decisions

- Which savings accounts should you choose:
 - 5.25% with daily compounding.
 - 5.30% with semiannual compounding.
- First account:
 - EAR = $(1 + .0525/365)^{365} - 1 = 5.39\%$
 - Excel: =EFFECT(0.525,365) = 5.39%
- Second account:
 - EAR = $(1 + .053/2)^2 - 1 = 5.37\%$
 - Excel: =EFFECT(0.53,2) = 5.37%

5-66

Computing APRs

- What is the APR if the monthly rate is .5%?
 - $.5\%(12) = 6\%$
- What is the APR if the semiannual rate is .5%?
 - $.5\%(2) = 1\%$
- What is the monthly rate if the APR is 12% with monthly compounding?
 - $12\% / 12 = 1\%$
- Can you divide the above APR by 2 to get the semiannual rate?
 - NO. You need an APR based on semiannual compounding to find the semiannual rate.

5-67

Computing EAR and APR

- Suppose you can earn 1% per month on \$1 invested today.
 - What is the APR? $1(12) = 12\%$
 - How much are you effectively earning?
 - $FV = 1(1.01)^{12} = 1.1268$
 - Rate = $(1.1268 - 1) / 1 = .1268 = 12.68\%$

Excel Solution

INCONV: NOM	= 12
C/Y	= 12
EFF	= 12.6825

= EFFECT(0.12,12)	
-------------------	--

Nominal rate		
Npery		
EFFECT=	0.12682503	

5-68

Chapter 5 Lecture - Discounted Cash Flow Valuation

Computing EAR and APR

• Suppose if you put it in another account, you earn 3% per quarter.

- What is the APR? $3(4) = 12\%$
- How much are you effectively earning?
 - $FV = 1(1.03)^4 = 1.1255$
 - Rate = $(1.1255 - 1) / 1 = .1255 = 12.55\%$

ICONV: NOM	= 12
C/Y	= 4
EFF	= 12.5509

Excel Solution

=EFFECT(0.12,4)

Nominal rate	12		
Npery	12		
		EFFECT=	0.125509

5-69

Frequency of Compounding

General Formula:

$$FV_t = PV_0 \left(1 + \frac{r}{m}\right)^{mt}$$

t: Number of Years

m: Compounding Periods per Year

r: Annual Interest Rate or APR

FV_t : FV at the end of Year t

PV_0 : PV of the Cash Flow today

5-70

Impact of Frequency

Ms. H has \$1,000 to invest for 2 years at an annual interest rate of 12%.

Annual $FV_2 = 1,000(1 + [.12/1])^{(1)(2)} = 1,254.40$

Semi $FV_2 = 1,000(1 + [.12/2])^{(2)(2)} = 1,262.48$

Quarterly $FV_2 = 1,000(1 + [.12/4])^{(4)(2)} = 1,266.77$

Monthly $FV_2 = 1,000(1 + [.12/12])^{(12)(2)} = 1,269.73$

Daily $FV_2 = 1,000(1 + [.12/365])^{(365)(2)} = 1,271.20$

5-71

Continuous Compounding

Continuous compounding implies $m \rightarrow \infty$

If -- r = interest rate per period (one year)

m = number of periods per year

t = number of years

$$FV_t = PV_0 (1 + r/m)^{mt} = PV_0 \{1 + 1/(m/r)\}^{mt}$$

$$= PV_0 \{[1 + 1/(m/r)]^{m/r}\}^{rt}$$

or

$$FV_t = PV_0 \{[1 + 1/(m/r)]^{m/r}\}^{rt}$$

5-72

Chapter 5 Lecture - Discounted Cash Flow Valuation

Solving the Equation

By definition $(1 + 1/n)^n$ as $n \rightarrow \infty$ is equal to $e = 2.7182\dots$

If interest is compounded continuously or $m \rightarrow \infty$,

$$\{1 + 1(m/r)\}^{m/r} = e \quad \text{Thus, } FV_t = PV(e)^{rt}$$

Example: $PV = 10000$ $r = 10\%$ $t = 5$

$$FV = 10000(e)^{.10(5)} = 10000 e^{0.5} = 10000 (1.6487) = 16487$$

Finding PV: $PV = FV / (e)^{rt}$

If you want to have 200,000 at retirement 10 years from now and you can invest at 10% interest, how much should you invest today?

$$PV = 200,000 / (e)^{.10(10)} = 200000 / e = 73578$$

5-73

Computing APRs from EARs

$$APR = m \left[(1 + EAR)^{1/m} - 1 \right]$$

M = number of compounding periods per year

Suppose you want to earn an effective rate of 12% and you are looking at an account that compounds on a monthly basis. What APR must they pay?

$$APR = 12 \left[(1 + .12)^{1/12} - 1 \right] = .1138655 \text{ or } 11.39\%$$

Excel Solution

=NOMINAL(0.12,12)

Effect rate	0.12	
Npery	12	
	NOMINAL=	0.113865515

ICONV: EFF	= 12
C/Y	= 12
NOM	= 11.3866

5-74

Computing Payments with APRs

- Suppose you want to buy a new computer.
- The store is willing to allow you to make monthly payments.
- The entire computer system costs \$3,500.
- The loan period is for 2 years.
- The interest rate is 16.9% with monthly compounding.
- What is your monthly payment?

=PMT(0.0140833,24,3500,0)

Excel Solution

Rate	.169/12	0.014083	
Nper		24	
PV		3500	
	PMT		(\$172.88)

5-75

Future Values with Monthly Compounding

Suppose you deposit \$50 a month into an account that has an APR of 9%, based on monthly compounding. How much will you have in the account in 35 years?

Excel Solution

=FV(0.0075,420,-50,0)

Rate	.09/12	0.0075	
Nper		420	
Pmt		-50	
	FV =		\$147,091.51

420	N	(35*12)
0.75	I/Y	(9/12)
0	PV	
-50	PMT	
CPT FV = 147,089.22		

5-76

Chapter 5 Lecture - Discounted Cash Flow Valuation

Present Value with Daily Compounding

You need \$15,000 in 3 years for a new car. If you can deposit money into an account that pays an APR of 5.5% based on daily compounding, how much would you need to deposit?

Excel Solution

=PV(0.00015,1095,0,15000)

1095	N (3*365)
.015068493	I/Y (5.5/365)
0	PMT
15,000	FV
	CPT PV = -12,718.56

Rate	.055/365	0.000151		
Nper		1095		
FV		15000		
			PV =	(\$12,718.56)

5-77

Pure Discount Loans

- Treasury bills are excellent examples of pure discount loans.
 - Principal amount is repaid at some future date
 - No periodic interest payments
- If a T-bill promises to repay \$10,000 in 12 months and the market interest rate is 7 percent, how much will the bill sell for in the market?

Excel Solution

PV(.07,1,0,10000)

1 N; 10,000 FV; 7 I/Y; CPT PV = -9345.79

Rate	0.07		
Nper	1		
FV	10000		
		PV =	(\$9,345.79)

5-78

Amortized Loan with Fixed Payment Example

- Each payment covers the interest expense plus reduces principal
- Consider a 4-year loan with annual payments. The interest rate is 8% and the principal amount is \$5000.
 - What is the annual payment?

Excel Solution

=PMT(0.08,4,5000,0) = 1509.60

Rate	0.08		
Nper	4		
Pv	5000		
		PMT	(\$1,509.60)

5-79

Amortized Loan with Fixed Payment - Example

Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	\$ 5,000.00	\$ 1,509.60	\$ 400.00	\$ 1,109.60	\$ 3,890.40
2	\$ 3,890.40	\$ 1,509.60	\$ 311.23	\$ 1,198.37	\$ 2,692.03
3	\$ 2,692.03	\$ 1,509.60	\$ 215.36	\$ 1,294.24	\$ 1,397.79
4	\$ 1,397.79	\$ 1,509.60	\$ 111.82	\$ 1,397.79	\$ -
Totals		\$ 6,038.40	\$ 1,038.42	\$ 5,000.00	

Interest Paid = Beginning Balance * Rate (8%)

Principal Paid = Total Payment - Interest Paid

Ending Balance = Beginning Balance - Principal Paid

5-80