CHAPTER 11 LECTURE – PRODUCTION, OUTPUT AND COSTS

Production Functions

A *production function* is a formal mathematical relation that describes the efficient process of transforming inputs into outputs. The word efficient is in the definition because embedded in the concept of the production function is the notion that firms will want to squeeze the maximum output from a given set of inputs.

A simple production function describes the process of transforming a set of inputs K, L, etc. into a quantity Q of output (goods or services): Q = Q (K, L etc.). We assume technology is fixed.

Production Decisions in the Short – Run

The short run is defined as the period in which there are fixed factors of production

<u>Variable and fixed factors of production</u> – variable factors are the inputs a manager can adjust to alter production. Fixed factors are the inputs the manager cannot adjust for a period of time

Usually capital (K) is fixed (denoted by $\overline{K}$) in the short run and labor (L) is variable, so we can rewrite the production function as, Q = f ($\overline{K}$, L)

Formal Definitions

- The short run is a time frame in which the quantities of some factors of production are fixed. The fixed factors include the firm's management organization structure, level of technology, buildings and large equipment. These factors are called the firm's plant.

- The long run is a time frame in which the quantities of *all* factors of production can be varied. Long-run decisions are not easily reversed so usually a firm must live with the plant size that it has created for some time. The past cost of buying a plant that has no resale value is called a sunk cost.

Total, Marginal, and Average Product

Total product (TP) is the entire output of the production process, and often denoted as the Q, or quantity of output.

Marginal product (MP) of a particular factor of production, for example labor, is defined to be the change in output (Q) resulting from a one-unit change in the input (ex: one more hour worked, or one more worker employed). If again we use the symbol Δ to mean 'change in', then we have:

$MP_X = \dfrac{\Delta TP}{\Delta X}$ = where X is a particular input, such as labor, and Q is output of the final good or service produced. Marginal product is the slope of total product.

We generally talk about labor so the marginal production of labor is $MP_L = \frac{\Delta TP}{\Delta L}$

Average product (AP) is the average amount of output produced with each unit of input:

$AP_X = \frac{TP}{X}$ Again, look at labor $AP_L = \frac{TP}{L}$ Keep in mind that TP = Q so we can also define $MP_L = \frac{\Delta Q}{\Delta L}$ and $AP_L = \frac{Q}{L}$

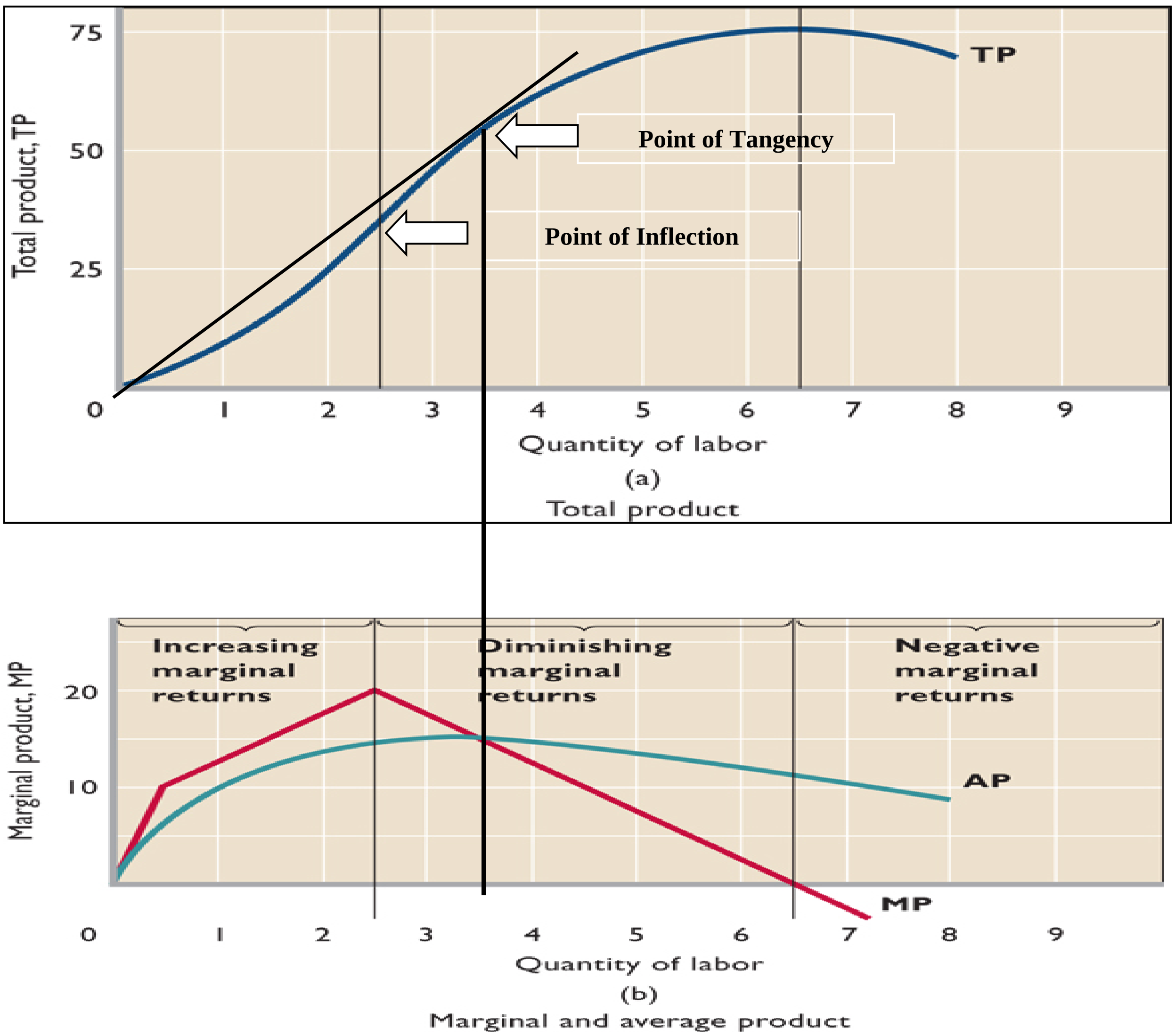
Total Product, Average Product and Marginal Product Relationships

Labor (Ceteris Paribus)	Total Product (Q)	Marginal Product of Labor	Average Product of Labor
0	0		
1	10	10	10
2	22	12	11
3	30	8	10
4	36	6	9
5	40	4	8
6	42	2	7
7	42	0	6

You can plot this.

Total, Marginal, and Average Product

(1) Units of the Variable Resource (Labor)	(2) Total Product (TP)	(3) Marginal Product (MP), Change in (2)/ Change in (1)		(4) Average Product (AP), (2)/(1)
0	0			—
1	10	10	Increasing marginal returns	10.00
2	25	15		12.50
3	45	20		15.00
4	60	15	Diminishing marginal returns	15.00
5	70	10		14.00
6	75	5		12.50
7	75	0	Negative marginal returns	10.71
8	70	−5		8.75



(a)
Total product

(b)
Marginal and average product

THE RELATIONSHIP BETWEEN TOTAL PRODUCT, MARGINAL PRODUCT AND AVERAGE PRODUCT

Diminishing Marginal Returns

Very generally, in the short run (when, for example, the production facility is fixed in size) there are two intuitive processes at work:

Gains in marginal productivity as we add more of a variable factor of production (ex: workers) due to specialization

Declines in marginal productivity as we add more of a variable factor of production due to congestion in the fixed factor (ex: a kitchen).

The concept of ***diminishing marginal returns*** comes from empirical observation of actual production processes. The idea is intuitive: as more and more of a variable factor (X) is combined with a fixed factor (Y), the marginal productivity of the variable factor, $\Delta Q/\Delta X$, eventually will decline. The central motivation for diminishing marginal returns to a variable factor is that the fixed factor will eventually become *congested* with the variable factor. As we keep adding more and more kitchen employees to a kitchen of fixed capacity, the space will become congested with kitchen workers and eventually there will not be enough capital (ex: pots, pans, space on the cooker, utensils, ovens) for each additional worker to be as productive as the previous worker added. Moreover, it is possible that the congestion can become so acute that hiring an additional worker will actually ***lower total product***, implying a negative marginal product.

In the general case we do not think that diminishing marginal returns sets in immediately because there frequently are economies of specialization, especially for labor as a variable factor. For example, as we initially hire a second kitchen worker, the two kitchen workers can now specialize -- one preps food ingredients, the other combines ingredients (cooks them) to make final meals.

Thus we might make a generalization for the purposes of illustration of the concept of marginal productivity that there are three stages of production:

Stage 1: Increasing marginal returns
Stage 2: Diminishing marginal returns
Stage 3: Negative marginal returns

We have shown in the previous diagrams the relationships between total product and the average and marginal product of the variable input (labor in this case)

1. When TP is at a maximum, $MP_L = 0$.
2. When the AP_L is increasing, then $MP_L > AP_L$.
3. When AP_L is at a maximum, $MP_L = AP_L$.
4. When the AP_L is declining, then $MP_L < AP_L$.

CAN YOU PROVE THESE STATEMENTS?

THE COSTS OF PRODUCTION

We can review costs again

The ***opportunity cost*** of an asset (or, more generally, of a choice) is the highest valued opportunity that must be passed up to allow current use.

Explicit costs are expenses for which one must pay with cash or equivalent. Because a cash transaction is involved, they are relatively easily accounted for in analysis.

Implicit costs do not involve a cash transaction, and so we use the opportunity cost concept to measure them. This analysis requires detailed knowledge of alternatives that were not selected at various decision points. Relevant here are the opportunity cost of the firm's assets and cash, and of the owner's time invested in the firm.

Sunk costs are those parts of the purchase cost that cannot later be salvaged or modified through resale or other changes in operations. Image advertising for a new product is a classic example of a sunk cost, as is an option or investment in assets whose value is specific to a particular situation. Sunk costs reflect *commitment*, or irreversibility, and so are not a part of incremental analysis.

***Accounting costs*:** measure historical costs, or costs actually paid.

Economic costs measure opportunity costs, or the cost in terms of the best forgone alternative.

Normal Profit - minimum level of profit required to keep the factors of production in their current use in the long run.

Also looked at as the minimum profit necessary to attract and retain suppliers in a perfectly competitive market). Only a normal profit could be earned in such markets because, if profit was abnormally high, more competitors would appear and drive prices and profit down. If profit was abnormally low, firms would leave the market and the remaining ones would drive the prices and profit up. Markets where suppliers are making normal profits will neither expand nor shrink and will, therefore, be in a state of long-term equilibrium. Normal profit typically equals opportunity cost.

Economic profit arises when its revenue exceeds the total (opportunity) cost of its inputs, noting that these costs include the cost of equity capital that is met by "normal profits."

A business is said to be making an **accounting profit** if its revenues exceed the accounting cost the firm "pays" for those inputs. Economics treats the normal profit as a cost, so when deducted from total accounting profit what is left is economic profit (or economic loss).

Short-Run and Long-Run Costs

In microeconomics and managerial economics, the ***short run*** is the decision-making period during which at least one input is considered fixed. The fixed input is commonly considered to be some aspect of capital, such the production facility, but may also be a normally variable input that is fixed because of production technology requirements, or a contractual commitment (e.g., a facility lease) related to production. So when one refers to short-run analysis, the analysis is focused on a planning period in which some input is fixed and others are variable, and the manager is selecting levels of variable input and production output to optimize given the constraint of the fixed input.

In contrast, the economic ***long run*** is a planning horizon that looks beyond current commitments to a future period in which all inputs can be varied. A typical long-run analytical problem is the decision of whether to adjust capacity, seek a larger (or smaller) facility, to change product lines, or to adopt a new technology.

Short Run Cost Curves

In order to examine a firm's costs, we need to know its optimal input mix and the prices of the inputs. Then we can define the important cost concepts:

Total Cost = TC **Total Fixed Cost** = TFC **Total Variable Cost** = TVC

TC = TFC + TVC

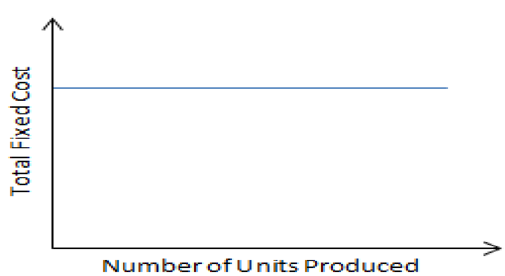
Average Total Cost $= ATC \text{ or } AC = \frac{TC}{Q}$ **Average Variable Cost** $= AVC = \frac{TVC}{Q}$

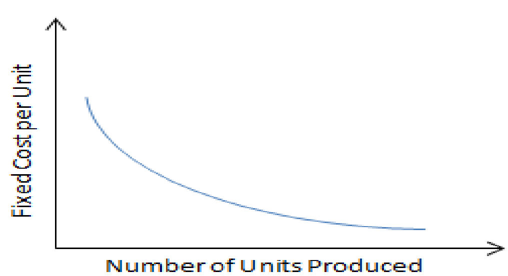
Average Fixed Cost $= AFC = \frac{TFC}{Q}$ **Marginal Cost** $= MC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta TVC}{\Delta Q}$

Total Cost is made up of two components in the short run: Total Fixed Cost and Total Variable Cost:

TC = TFC + TVC so $\frac{TC}{Q} = \frac{TFC}{Q} + \frac{TVC}{Q}$ = ATC = AFC + AVC

Now we need to talk about **Total Fixed Costs (TFC)** and **Average Fixed Costs (AFC)**. The Fixed Costs do not change as output varies in the short run, so they may be represented as a horizontal line. Average Fixed Costs continuously declines.:

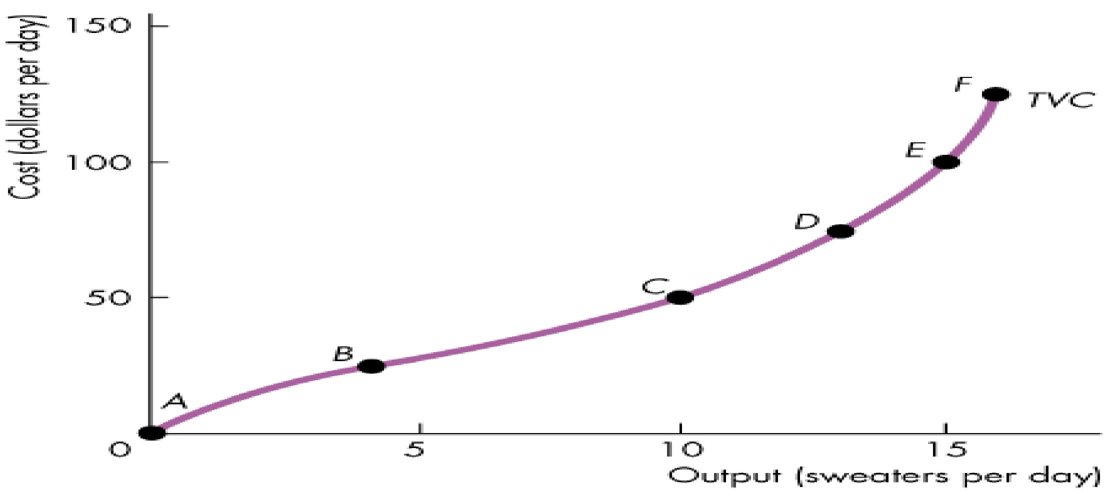




Total Variable Costs are the costs associated with hiring various levels of variable inputs in order to vary the rate of output in the short run."

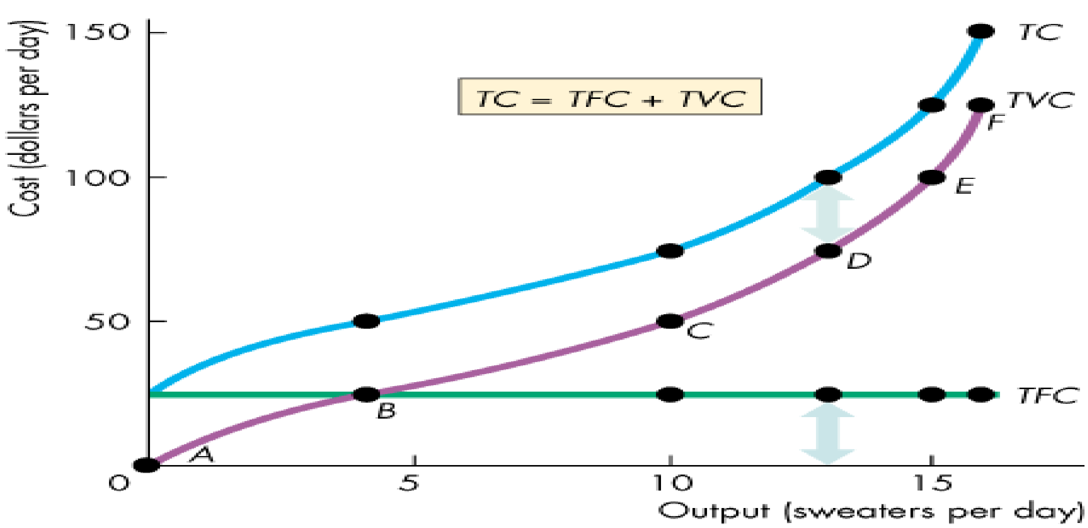
It is easy to translate the quantity of the variable input into total variable cost (TVC): just multiply by the price of the input - the shape doesn't change. However, now we would like to express TVC as a function of the output of the product. This is easy: just flip the axes and we have the TVC curve:

TVC = P_L x L



Finally, recall that the firm's Total Cost of producing output (TC) is the sum of its Total fixed cost (TFC) and it's Total Variable Cost (TVC):

TC = TFC + TVC



But, what the firm really needs to know is how the costs are distributed across the individual units of outputs and how they change when the level of output is increased or decreased. Thus, we want to look at the shapes of Average Total Cost (ATC), Average Variable Cost (AVC), and Marginal Cost (MC). Average Fixed Cost (AFC) you can derive on your own.

Average Variable Cost (AVC)

AVC is the slope of the line from the origin to the point on the TVC function. This slope is a direct result of the law of diminishing marginal returns.

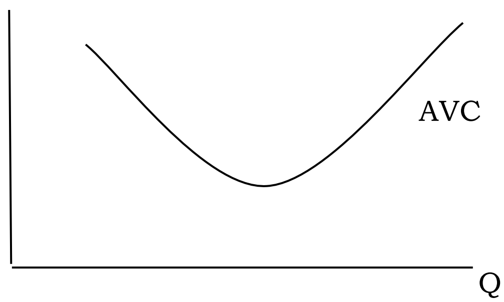
Now assume labor is only input so TVC = P_L x L For simplicity assume $P_L = 1$

Thus, TVC = L $AVC = \frac{TVC}{Q} = \frac{L}{Q}$ But $AP_L = \frac{TP}{L} = \frac{Q}{L}$

So $AVC = \frac{1}{AP_L}$. As the AP_L falls, AVC rises and

as AP_L rises, AVC falls. If AP_L is constant, AVC is constant.

$AVC = \frac{TVC}{Q}$

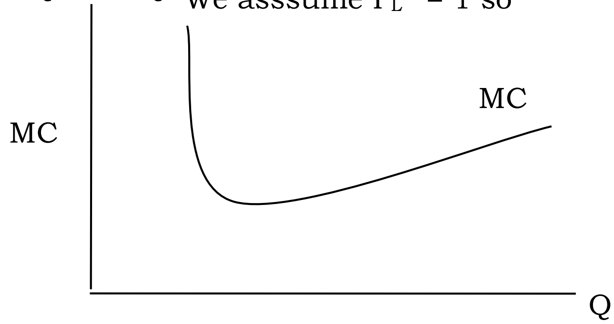


Marginal Cost (MC)

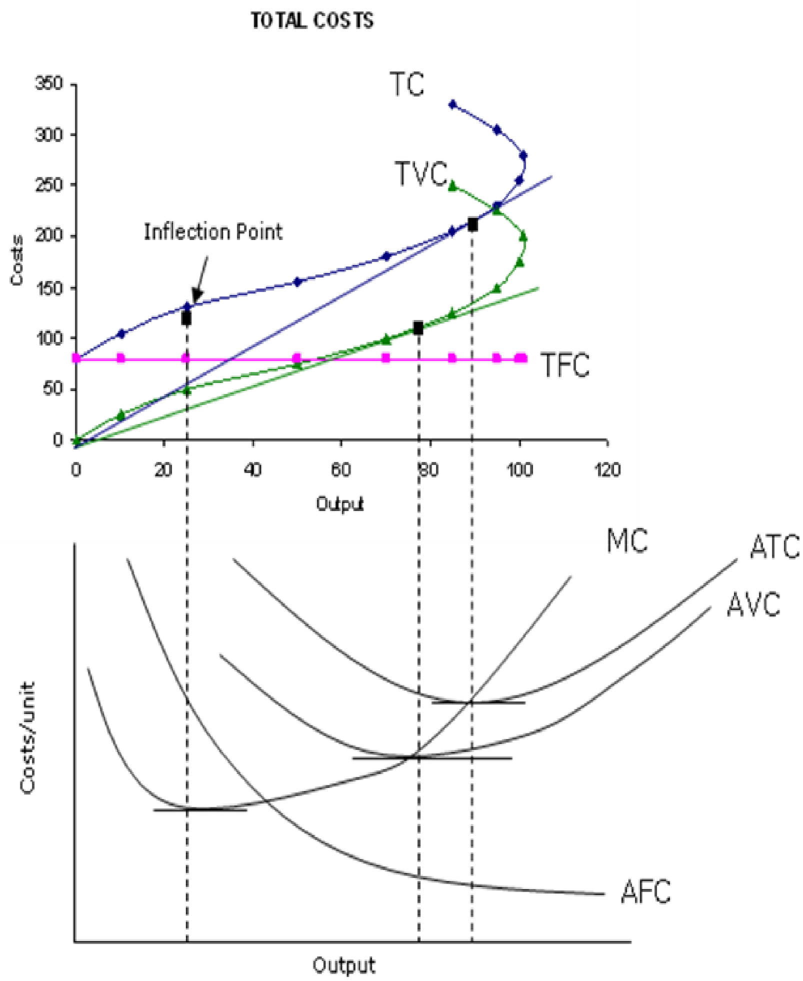
MC is the slope of TC and TVC. The shape is a direct result of the law of diminishing marginal returns.

From before TVC = P_L x L For simplicity assume $P_L = 1$ so TVC = L

$$MC = \frac{\Delta TVC}{\Delta Q} = \frac{\Delta L}{\Delta Q}$$ We asssume $P_L = 1$ so $$MC = \frac{\Delta TVC}{\Delta Q} = \frac{\Delta L}{\Delta Q} = \frac{1}{MP_L}$$



Relationship Among Cost Curves Once Again



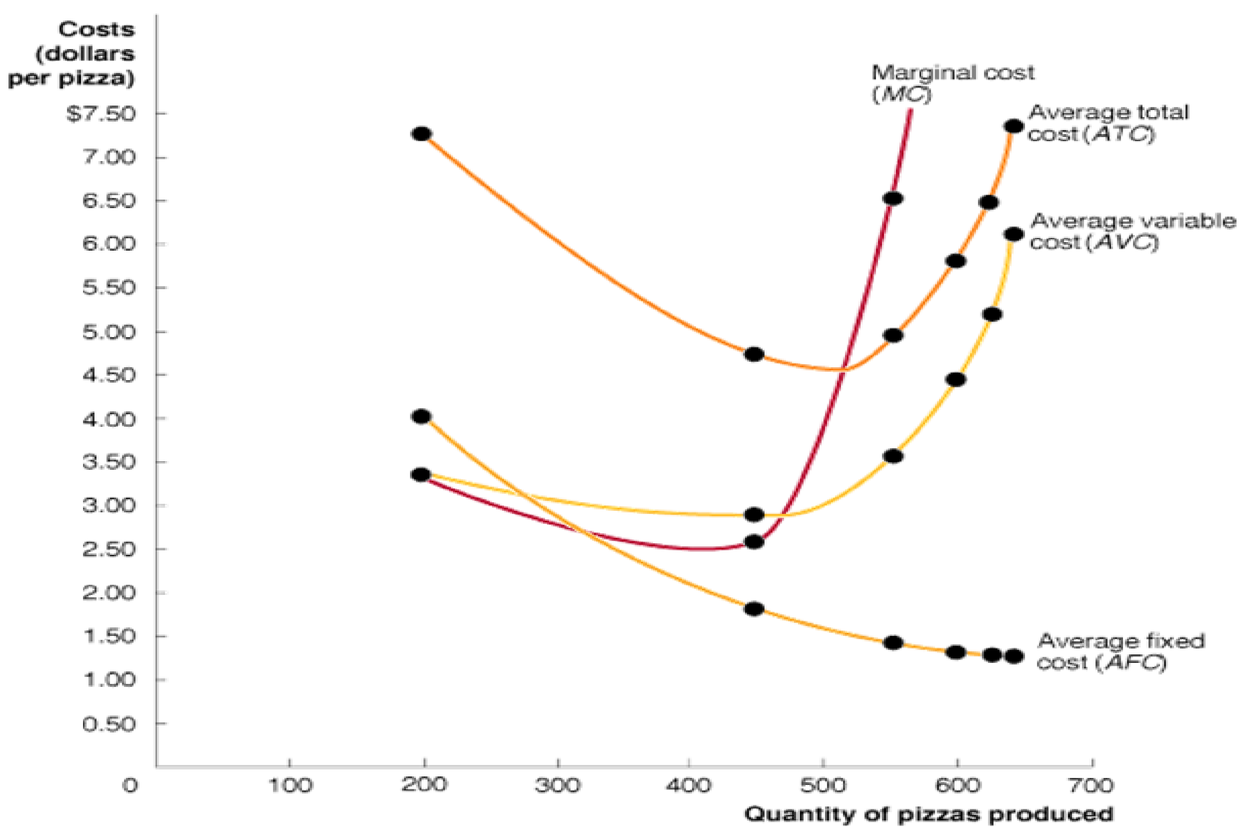
The Relationships Among Short-Run Cost Curves

1. *AFC* continuously declines and approaches both axes asymptotically.
2. *AVC* initially declines reaches a minimum and then increases.
3. When *AVC* is at a minimum it is equal to *MC*.
4. *ATC* initially declines reaches a minimum and then increases.
5. When *ATC* is at a minimum it is equal to *MC*.
6. *MC* is less than *AVC* and *ATC* when both curves are declining.
7. *MC* is greater than *AVC* and *ATC* when these curves are increasing.
8. *MC* equals *AVC* and *ATC* when both curves reach their minimum values.

MC passes through minimum of AC.

Numerical Example Short-Run Cost

Quantity of workers	Quantity of ovens	Quantity of Pizzas	Cost of Ovens (fixed cost)	Cost of workers (variable cost)	Total Cost of Pizzas	*ATC*	*AFC*	*AVC*	*MC*
0	2	0	$800	$0	$800	–	–	–	–
1	2	200	800	650	1,450	$7.25	$4.00	$3.25	$3.25
2	2	450	800	1,300	2,100	4.67	1.78	2.89	2.60
3	2	550	800	1,950	2,750	5.00	1.45	3.54	6.50
4	2	600	800	2,600	3,400	5.67	1.33	4.33	13.00
5	2	625	800	3,250	4,050	6.48	1.28	5.20	26.00
6	2	640	800	3,250	4,050	6.48	1.28	5.20	26.00
6	2	640	800	3,900	4,700	7.34	1.25	6.09	43.33



Cost Functions in the Long-Run

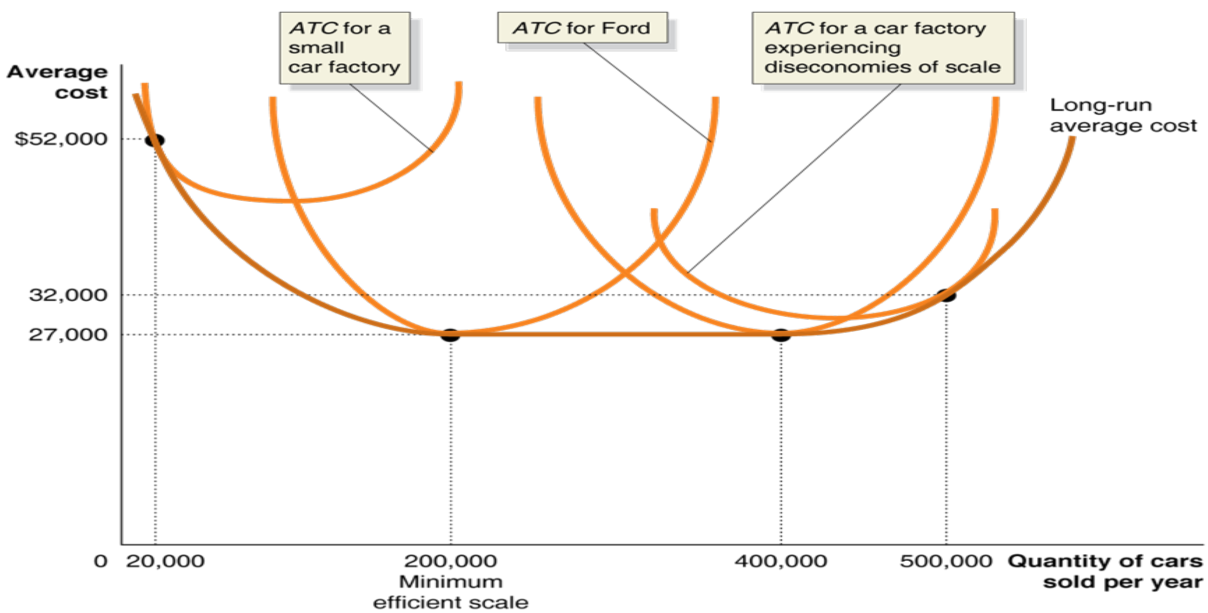
What we will look at now is the long run -- this where a firm can adjust ALL inputs in an optimal manner. In consumer theory -- indifference curves -- the analogous problem is how you adjust in response to the prices of goods changing. If the prices of X and Y you adjust the combination of goods that you buy moving to another optimal bundle. Well, a firm does the same thing, it's buying labor and capital and if the prices of those things change, then a firm readjusts how many people it hires, how many computers to use, etc.

Long-Run Average Cost as the Envelope of Short-Run Average Cost Curves

Let's take a firm that has the following production function: Q = f (K, L). And it can buy all the labor it wants at P_L and all the capital it wants at P_K.

Therefore, total costs are equal to: $TC = P_L L + P_K K$.

The first curve we'll talk about is the Average cost curve; obviously ATC = TC/Q.



We now turn to the concept of Economies and Diseconomies of Scale. Remember we had the concept of diminishing marginal product of labor -- in that situation, the only thing that changed was labor, we hold the amount of capital constant. EOS deals with the situation of what happens to output, when we change everything.

We know we can have:

Increasing Returns to Scale (Economies of Scale): %ΔOutput>%ΔInputs

Constant Returns to Scale: %ΔOutput=%ΔInputs

Decreasing Returns to Scale (Diseconomies of Scale): %ΔOutput<%ΔInputs

Consider what happens to short run costs as we change K. As K increases, we get new sets of SR cost curves. They will shift to the right, as shown below, indicating that we can produce a higher level of output as we increase the size of our plant. Typically, the minimum point of the SRAC curve will decrease as K increases at first, then it may remain relatively constant for a while, and finally it will begin increasing as K increases. This reflects economies of scale.

In the long run, how should we select the quantity of K and L we want to use to produce a given level of output? Show.

If the LRAC is downward sloping we have increasing returns to scale.

Why might we have increasing returns to scale? Specialization, technology becomes more efficient as scale increases, large initial start-up or administrative costs that get spread more efficiently as scale increases.

If the LRAC is upward sloping we have decreasing returns to scale.

Why might we have decreasing return to scale? Exhaust specialization and more efficient technology, administrative burden increases.

If the LRAC is horizontal we constant returns to scale (CRS). In this case, LRAC remain constant as output increases (if outputs and inputs both double, average total costs will not change).

The lowest level of output at which all economies of scale are exhausted is known as the **minimum efficient scale**.